Feasibility Analysis of Contingent Capital Provisions: The Case of CatEPuts

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Abstract

This study investigates how contract provisions affect the valuation of an insurer's contingent capital and highlights the divergent perspectives of sellers, the insurer's debtholders, and equity holders. Using real-world `CatEPut` contracts, we uncover the pivotal role of early exercise and net-worth provisions in enhancing contract tradability and the insurer's firm-levered value, mainly through bankruptcy costs. Our novel model integrates catastrophe risk, insolvency risk, capital structure, emergency capital injection, and equity dilution. Our analyses provide a theoretical foundation for the design of general contingent capital contracts, bridging a critical gap in current research.

Keywords: Contingent Capital, Catastrophe Risk, CatEPut, Trade-off Theory, Contract Provision

1 Introduction

Climate and catastrophe risks have received increasing attention within the insurance sector over the past decades. According to Swiss Re (2018), catastrophic events in 2017 resulted in a record-breaking \$377 billion in total economic losses and \$144 billion in insured losses. Looking ahead, Gates (2021) expressed his concerns, stating:

> *In the next decade or two, the economic damage caused by climate change will likely be as bad as having a COVID-sized pandemic every 10 years.*

Catastrophes not only lead to substantial losses but also undermine the fundamental insurance principles of the law of large numbers, thereby impeding insurers' ability to effectively diversify risk (Froot, 2001; Froot and O'Connell, 2008). These challenges surpass the capacities of traditional reinsurance

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approaches and adversely impact insurers' financial stability. Consequently, the practice of transferring catastrophe risks to the capital markets using contingent capital facilities has emerged as a modern solution within the insurance sector.[1]

A contingent capital facility grants the holder the right to issue new debt or equity after a specified triggering event within a predefined period at a predetermined issue price. In 1996, RLI Corp's issuance of `CatEPut`[2] marked the insurance industry's first contingent equity transaction, following RLI's substantial losses resulting from the 1994 Northridge earthquake (Culp, 2011). Under this arrangement, RLI can issue convertible non-voting preferred shares, valued at up to $50 million, as a timely capital injection if a major Californian earthquake is triggered within three years. Another compelling case involves SCOR, which encountered significant losses from calamities in Australia, New Zealand, and Japan in 2011 and subsequently issued shares worth up to €75 million to UBS due to the `CatEPut` trigger (Culp, 2011). Notably, the Artemis news website reported that SCOR recently initiated its fourth renewal of contingent capital facilities for the period spanning 2023 to 2025, with a capital capacity of up to €300 million, primarily oriented towards the transfer of catastrophe and extreme mortality risks. These real-world transactions solidly affirm the practicality of contingent capital facilities as a balance sheet recovery mechanism, effectively bolstering insurers' financial stability and aiding risk transfer in the event of a catastrophe.[3]

Although contingent capital facilities play a critical role in catastrophe risk management and solvency maintenance, it is imperative to recognize both their costs and benefits, as well as potential conflicts of interest among the parties involved. However, the valuation of insurers' contingent capital, especially contingent equity, poses challenges due to their intricate contract designs.[4] These facilities essentially operate as dual-trigger put options, contingent on two conditions: the first trigger occurs when catastrophe losses accumulate to a predetermined level, and the second condition is triggered when the stock price of the contract buyer (typically an insurer) falls below the strike price (i.e., the predetermined issue price).[5] The complexity arises as the insurer's stock price both determines the option trigger and is influenced by the post-trigger capital injection, leading to an endogeneity problem (Lo et al., 2013; Sundaresan and Wang, 2015; Glasserman and Nouri, 2016). This problem intensifies in cost-benefit analysis, as financing costs reduce the insurer's post-purchase asset value (along with other claimholders' values), whereas the emergency capital injection strengthens the post-

[1] For more details on other insurance-linked securities, see Härdle and Cabrera (2010), Lakdawalla and Zanjani (2012), and Zhao and Yu (2020) for catastrophe bonds, and Braun (2011) and Lo et al. (2021) for catastrophe swaps.

[2] The trademark `CatEPut`, originating from catastrophe equity puts, is registered with the insurance broker Aon and stands as one of the most famous contingent equity products in the insurance industry.

[3] Contingent capital facilities have captured attention not only within the insurance domain but also in banking, particularly following the 2008 financial crisis (see Hilscher and Raviv, 2014; Pennacchi et al., 2014; Sundaresan and Wang, 2015; Chen et al., 2017; Pennacchi and Tchistyi, 2019).

[4] Lo et al. (2013) argue that contingent debt, such as contingent surplus notes, shares a contract design that is similar to but simpler than contingent equity. Hence, this study concentrates on contingent equity.

[5] Within the banking industry, the catastrophe trigger in contingent capital is substituted by the market trigger (see Sundaresan and Wang, 2015; Glasserman and Nouri, 2016; Chen et al., 2017; Pennacchi and Tchistyi, 2019).

exercise values. In addition to the price endogeneity, timely capital injection is expected to alleviate the insurer's insolvency risks in the face of catastrophe; nevertheless, the issuance of new shares dilutes the value of existing shareholders (Koziol and Lawrenz, 2012; Pennacchi et al., 2014; Sundaresan and Wang, 2015; Chen et al., 2017; Liu et al., 2022). Note that the gains and losses of contingent equity buyers and sellers vary due to the altered default likelihood caused by emergency capital injections, leading to divergent valuation perspectives.[6]

The above-mentioned valuation complexities are amplified by considering real-world contract provisions that influence the rights and obligations of the parties involved.[7] Upon examining the contract specifics in the financial statements of contingent equity buyers such as RLI Corp, Horace Mann Educators Corporation, LaSalle Re Educators Corporation, and the Trenwick Group, we observe a common structure. These real-world contracts consistently take the form of multi-year American-style options and include a net-worth provision.[8] The American-style contract grants the buyer the right to immediately issue equity once the contract is triggered, even before its scheduled expiration. This early exercise provision usually increases the time value of put options. Conversely, the net-worth provision imposes a condition on the insurer: it cannot exercise the option if its GAAP net worth falls below a pre-agreed threshold.[9] This provision reduces the contract value and acts as a knock-out barrier option to protect the seller from the obligation of injecting emergency funds into an insurer that might be struggling financially and unable to continue its operations. By including the two provisions in our study, we seek to investigate the tradability of insurers' contingent capital and its impact on claimholder values from both the buyer's and seller's viewpoints.

To tackle these challenges, we develop a novel twin-tree model with jumps (`TTMJ`) to value contingent equity, analyzing the changes in claimholders' values after the transaction and exploring whether the contract design benefits both parties. This model integrates several concepts from the literature. First, inspired by Liu et al. (2016)'s forest method, we construct two trees to simulate asset value dynamics before and after contingent capital exercise. Resembling parallel universes, this twin-tree structure effectively addresses capital injection and equity dilution at the option trigger by transferring between trees, reflecting changes in the asset value and capital structure. Second, we incorporate negative jumps through a compound Poisson process into the insurer's asset value to capture cumulative catastrophe losses (Bakshi and Madan, 2002). Third, in line with the structural models of Sundaresan and Wang

[6] Specifically, mitigating the insurer's insolvency risks extends the insurer's life, granting additional tax-shield benefits to insurers which are not paid by contingent equity sellers. A win-win scenario arises when the contingent equity purchase price surpasses the seller's minimum required price yet remains below the buyer's maximum acceptable price.

[7] Analyzing the impact of real-world contract provisions parallels the approach taken by Pennacchi and Tchistyi (2019), who introduce a real-world perpetuity feature for bank contingent convertibles to address the problem of missing or multiple equilibria in stock prices identified in the banking literature.

[8] Many studies propose sophisticated models for the valuation of contingent equity; however, the majority of these studies concentrate on the valuation of European-style contracts and tend to overlook net-worth provisions (see Cox et al., 2004; Jaimungal and Wang, 2006; Chang et al., 2011).

[9] In the case of the RLI contract, it can only be exercised if the loss does not reduce equity to less than \$55 million, as stated in its 1997 10-K annual report. Similarly, the contracts of Horace Mann, LaSalle, and Trenwick include net-worth provisions of \$175 million, \$175 million, and \$125 million, respectively.

(2015) and Chen et al. (2017), among others, the insurer insolvency is endogenously determined in our model, and the capital injection from contract exercise can directly impact the insurer's capital structure and default risk.[10] Fourth, the price endogeneity noted by Lo et al. (2013), Sundaresan and Wang (2015), and Glasserman and Nouri (2016) can be solved by a simple iterative algorithm that repeatedly applies our `TTMJ`. Finally, the `TTMJ` can address the early exercise provision (Lo et al., 2013; Wang and Dai, 2018) and the net-worth provision (Doherty, 1997) to examine their impact on the tradability of `CatEPut` contracts.

In our quantitative analyses, we retrieve RLI's `CatEPut` contract details and RLI's financial statuses from its financial report as well as U.S. catastrophe data to rebuild the scenario in which RLI decides to purchase the contract. Since our structural model incorporates various components, including company assets, debt, equity value, bankruptcy costs, and tax benefits, we utilize the trade-off theory (see Kraus and Litzenberger, 1973) both to affirm its validity and to analyze the changes in claimholders' values due to `CatEPut` purchases. However, in cases where catastrophe losses could exceed the insurer's asset value, as highlighted by Lakdawalla and Zanjani (2012), the traditional trade-off theory becomes inadequate.[11] Hence, we introduce a revised trade-off theory, enabling it to address situations such as government bailouts or the financial burden shouldered by acquirers when taking over insolvent insurance firms.[12] These risks of reference entities have not been thoroughly analyzed in previous studies, and our model addresses this gap by quantifying insolvency phenomena overlooked in the traditional trade-off theory.

The main findings of this paper can be summarized as follows. First, when the buyer has high leverage, the inclusion of a net-worth provision significantly reduces the likelihood of capital injections, thereby lowering the seller's minimum required price for the contract. Surprisingly, this provision has received limited attention in the pricing literature. Second, the inclusion of both early exercise and net-worth provisions can lead to mutually beneficial outcomes for sellers, insurers' equity holders, and debtholders. The early exercise provision facilitates timely capital injections, lowering the likelihood of the insurer's default and allowing the insurer to accept a higher price. Meanwhile, the net-worth provision prevents ineffective capital injections that primarily benefit debtholders during liquidation, leaving equity holders without compensation. Considering perspectives from both parties, we highlight the crucial role of both provisions in enhancing contract tradability, making contingent capital transactions more likely to occur. Third, changes in insurers' firm-levered value due to the `CatEPut` purchase are shown to be positive (negative) for contracts with (without) both provisions, and these changes

[10] Numerous studies (e.g., Cox et al., 2004; Jaimungal and Wang, 2006; Chang et al., 2011) adopt the reduced-form approach to value the insurer's contingent capital, modeling catastrophe risk as jumps in stock returns. However, when acquiring contingent equity, this approach does not faithfully model the trade-offs between the costs and benefits of the insurer's capital structure due to capital injections and dilution.

[11] A notable example is the insolvency of eight property and casualty insurers caused by Hurricane Andrew in 1992. See the news "Hurricane Andrew slammed into South Florida on August 23 and 24, 1992 – and changed the insurance industry forever.".

[12] For example, the government took over Access Home Insurance Company and State National Fire Insurance Company after Hurricane Ida in 2021. See the news with title "Louisiana Seeks Take-Over of Failing Insurers After Ida".

are primarily driven by changes in debt value (equity value). Last, our analysis, grounded in the revised trade-off theory, reveals that the changes in firm-levered value are predominantly influenced by changes in bankruptcy costs across all contract types. This analysis elucidates the mechanism behind the enhancement of contract tradability by early exercise and net-worth provisions, reflecting their role in reducing the insolvency risk borne by insurers.

This study makes a four-fold contribution to the existing literature. First, we highlight the valuation disparities among contract parties, providing a channel to analyze the contract tradability. In contrast, previous studies (Cox et al., 2004; Jaimungal and Wang, 2006; Chang et al., 2011) focus on the seller's viewpoint to evaluate the present value of emergency capital injections, which may not match the buyer's gain. Second, our model elucidates the rationale behind provisions commonly found in real-world contracts but often overlooked in the literature (Chang et al., 2011; Lo et al., 2013; Wang and Dai, 2018). We find that including both early-exercise and net-worth provisions simultaneously benefits all claimholders, offering a new perspective on their role in contingent capital design. Third, we expand the trade-off theory (Kraus and Litzenberger, 1973) to include the possibility of government bailouts or losses incurred by acquirers when taking over insolvent insurance companies, a critical consideration for contract buyers facing catastrophe risk. Finally, beyond our primary focus on `CatEPuts`, the generality and flexibility of our valuation techniques are applicable to assessing the pros and cons of analogous designs in other contingent claims (Sundaresan and Wang, 2015; Chen et al., 2017; Pennacchi and Tchistyi, 2019).

The remainder of this article is as follows. **Section 2** presents the model assumptions and techniques for the valuation and reviews the relevant literature. **Section 3** develops the `TTMJ` method, evaluates the contingent capital facilities from both the seller's and the buyer's perspectives, and revises the trade-off theory. **Section 4** analyzes how the contingent capital provisions affect the contract tradability and the changes in different parties' benefits. **Section 5** concludes this article.

2 Model

This section describes the model assumptions required to value a T-year contingent equity facility, a dual-trigger option that transfers the insurer's catastrophe risk to the capital markets. The first trigger condition is that the cumulative catastrophe losses suffered by the option buyer (usually the insurer) during the contract period must exceed a predetermined level L. To avoid such distress from deteriorating the insurer's financial status, a contingent equity facility grants the insurer the right to raise capital by issuing $\mathcal{N}_{\text{new}}$ new shares at a pre-agreed price K. Therefore, the second trigger condition is that the insurer's stock price is below the strike price K, inducing the insurer to exercise the option contract.

To analyze the impact of the contingent equity (or the injection of new shares issued) on the benefits of claimholders of the insurer, we evaluate contingent equity and corresponding equity and debt values by constructing a discrete-time tree model with time steps $\mathcal{T} \equiv \{0, \Delta t, 2\Delta t, \ldots, T\}$. This discrete-time framework also allows us to tackle the early exercise decisions of the contract holder since real-world `CatEPut` contracts are all American-style. Subsequently, we will introduce the settings of catastrophe losses, asset values, debt values and default conditions, and equity values as well as stock prices in **Sections 2.1**, **2.2**, **2.3**, and **2.4**, respectively.

2.1 Catastrophe Losses

Natural catastrophes are rare, unanticipated, and can cause extensive and severe damage, exposing insurers to potentially large claims. To characterize these features, Bakshi and Madan (2002) use a compound Poisson process to model the aggregate catastrophe losses.[13] They first identify the occurrence of catastrophic events through a Poisson process, followed by another independent random variable to determine the magnitude of catastrophe losses caused by each event.

Following previous studies, the occurrence of catastrophes in this study is determined by a Poisson process with probability $P(N_{t+\Delta t} - N_t = k) = \frac{e^{-\lambda \Delta t}(\lambda \Delta t)^k}{k!}$, $k = 0, 1, 2, \ldots$, where N_t denotes the cumulative number of catastrophes occurring up to time t with an initial value of $N_0 = 0$, and λ is the intensity parameter describing the expected number of catastrophes per year. If no catastrophe occurs during $(t, t + \Delta t]$, then $N_{t+\Delta t} = N_t$ and there are necessarily no catastrophe losses. However, if there is at least one catastrophe during this period, the magnitude of the aggregated loss can be modeled by a generalized Pareto distribution widely adopted in the literature, such as Hogg and Klugman (1983) and Härdle and Cabrera (2010). To reflect the discrete characteristic of our model, we use the discrete version of the Pareto distribution, i.e., the Zeta distribution, to model the magnitude of catastrophe losses (see Malamud et al., 1998; Pollett et al., 2007). Assume that the loss magnitude of the i-th catastrophe is Z_i units, where Z_i is a positive integer with a probability mass function of

$$P(Z_i = z) = \frac{z^{-s}}{\zeta_s}, \quad z \in \mathbb{N}. \tag{1}$$

Here, $s > 1$ is the shape parameter, and $\zeta_s = \sum_{k=1}^{\infty} k^{-s}$ is the Riemann zeta function. The loss unit can be arbitrarily defined to determine the distribution of Z_i to calibrate the catastrophe parameter s. For example, we consider the statistics provided by Swiss Re (2018) on loss magnitudes and set the loss unit to US\$10 billion.

Note that $Z_i \times \$10$ billion represents the global insured loss of the i-th catastrophe, while insurers' contingent capital facilities are typically triggered based on the accumulated losses suffered by the contract buyer. Thus, we estimate the loss of the i-th catastrophe suffered by the contract buyer as

[13] Compound Poisson jumps are also commonly used to model the value changes in the bank's assets (see Sundaresan and Wang, 2015; Chen et al., 2017), insurer liability (see Cummins, 1988; Duan and Yu, 2005; Gatzert and Schmeiser, 2008; Lo et al., 2013), and the share prices (see Jaimungal and Wang, 2006; Chang et al., 2011).

the buyer's market share m multiplied by the whole market loss, denoted by $L_i \equiv m \times Z_i \times \$10$ billion. For example, in 1996, the market share of RLI Corporation in California was approximately 0.18%, so the amount of RLI's losses caused by the i-th catastrophe is estimated to be $Z_i \times \$18$ million.

We set an upper bound for Z_i at a sufficiently large number, such as 15 in our subsequent experiments, for the following three reasons.[14] First, insurance policies generally specify a maximum indemnity amount, so the catastrophe loss suffered by an insurer is also bounded. Second, according to a report by Swiss Re, global insurance losses for natural catastrophes in 2021 are estimated to be about $105 billion, which is the sum of all catastrophes in a year.[15] Setting the cap at 15 implies that the maximum possible loss is $150 billion, which should be sufficient to cover the loss caused by one catastrophe. Third, the probability value of Equation (1) decays rapidly and does not affect our valuation and quantitative analyses even when we relax the upper bound.

2.2 Asset Value

Similar to the structural models of Sundaresan and Wang (2015) and Chen et al. (2017), we assume that the insurer's asset value consists of a log-normal diffusion process and a compound Poisson jump process to capture catastrophe losses. Therefore, the dynamic of the insurer's asset value V_t under our discrete-time framework is assumed to be:

$$V_{t+\Delta t} = V_t e^{(\mu_t - \frac{\sigma^2}{2})\Delta t + \sigma(W_{t+\Delta t} - W_t)} - (1-\tau)cD\Delta t - \sum_{i=0}^{\Delta N_t} L_i, \tag{2}$$

where μ_t is the time-varying drift term, σ is the volatility of the diffusion term, and W_t is a Wiener process. The second term of Equation (2) describes the coupon payment excluding tax benefits for each time period, where τ denotes the tax rate, c denotes the coupon rate, and D denotes the face value of the debt. The last term of Equation (2) shows the reduction in firm value due to catastrophe losses during the period $(t, t+\Delta t]$, as defined in Section 2.1, where ΔN_t represents $N_{t+\Delta t} - N_t$.

Modeling the impact of catastrophe losses on asset values is critical since catastrophes can deteriorate the insurer's financial status and even lead to insolvency.[16] We model this negative impact by downward jumps in firm value rather than in the log return to firm value, unlike much of the structural model literature, such as Sundaresan and Wang (2015) and Chen et al. (2017). Our concern is that modeling catastrophe losses through a jump term in log returns is more likely to generate minor (more significant) losses when the insurer has experienced a decline (rise) in asset value. This relationship contradicts Froot (2001), Cummins and Trainar (2009), and Ammar (2020), who argue that catastro-

[14] Technically, ζ_s is truncated to $\sum_{k=1}^{15} k^{-s}$ and $P(Z_i > 15)$ is set to zero.

[15] See the news with the title "Global insured catastrophe losses rise to USD 112 billion in 2021, the fourth highest on record, Swiss Re Institute estimates."

[16] For example, two property insurance companies—State National Fire Insurance Company of Baton Rouge and Access Home Insurance Company of New Orleans in Louisiana—entered insolvency after severe losses from Hurricane Ida in 2021[12].

phe risk is uncorrelated with a firm's financial risk. In contrast, the magnitude of catastrophe losses is not associated with the change in asset values if we model losses as jumps in firm values.

2.3 Debt Value and Default Boundary

A structural model pioneered by Merton (1974) views the value of a claim, say, the debt value, as a contingent claim on the insurer's asset defined in Equation (2). Debt holders can only recover part of their funds when the insurer becomes insolvent, and this paper defines the time point of insolvency using the first passage time model as $t^* = \inf\{t \geq 0 \mid V_t \leq B_t\}$. The default boundary at time t, denoted as B_t, is defined as the present value of the future coupon and debt principal repayments before maturity T. Formally, $B_t = \sum_{t<t_j\leq T} (1-\tau)cDe^{-r(t_j-t)} + De^{-r(T-t)}, \quad \forall\, t \leq T$, where r is the risk-free rate and t_j denotes the coupon payment date. In particular, the default boundary at maturity B_T is exactly the face value of the debt D.

When the insurer becomes insolvent at time t^*, debtholders receive the value of the insurer's after-liquidation assets, $(1-\alpha)V_{t^*}$, where the constant fraction α measures the liquidation cost (Leland, 1994; Collin-Dufresne and Goldstein, 2001; Sundaresan and Wang, 2015). We do not consider the recovery of future coupon payments because, as Collin-Dufresne and Goldstein (2001) suggest, claims on future coupon payments have the lowest priority. Therefore, the time-t debt value of a solvent insurer can be expressed as

$$D_t = \mathbb{E}_t^Q \left[\sum_{t<t_j\leq T} cDe^{-r(t_j-t)} \mathbf{1}_{\{t^*>t_j\}} + De^{-r(T-t)} \mathbf{1}_{\{t^*>T\}} + (1-\alpha)V_{t^*}e^{-r(t^*-t)} \mathbf{1}_{\{t^*\leq T\}} \right], \tag{3}$$

where $\mathbb{E}_t^Q[\cdot]$ is the expectation under the risk-neutral measure and $\mathbf{1}_{\{\cdot\}}$ is the indicator function. The first term in the expectation shows the present value of all future coupon payments before the insolvent event; the second term denotes the present value of the principal repayment given that the insurer is solvent; and the last term shows the funds recoverable at the point of insolvency. Although there is no closed-form formula for the debt value in Equation (3) under our model assumptions, this study proposes a numerical method to evaluate the values of debt and other claims, as detailed in Section 3.1.

Note that the debt value is related to the leverage ratio defined by $\eta \equiv D/V_0$ (see Leland, 1994). Specifically, the relation is implicitly embedded in Equation (3) of the indicator functions $\mathbf{1}_{\{t^*>T\}}$, $\mathbf{1}_{\{t^*>t_j\}}$, and $\mathbf{1}_{\{t^*\leq T\}}$. This relation also influences the valuation of tax benefits $\mathtt{TB}_t = \tau\mathbb{E}_t^Q \left[\sum_{t<t_j\leq T} cDe^{-r(t_j-t)} \mathbf{1}_{\{t^*>t_j\}} \right]$ and the bankruptcy costs $\mathtt{BC}_t = \alpha\mathbb{E}_t^Q \left[V_{t^*}e^{-r(t^*-t)} \mathbf{1}_{\{t^*\leq T\}} \right]$. Note that contract buyers receive tax benefits through reduced tax payments, which are not contributed by sellers. In addition, the option payoffs provided by the sellers might flow to bankruptcy costs instead of being received by the buyers. Our analyses capture the aforementioned inconsistency between the seller's payout and the buyer's gain and thus can produce different valuation results from buyers' and sellers' perspectives.

2.4 Equity Value and Stock Price

According to Black and Cox (1976) and Brockman and Turtle (2003), the equity value can be viewed as a down-and-out call option on the firm's asset value, with a strike price equal to the debt face value plus the coupon payment at maturity. Consequently, the equity value E_t can be expressed as

$$E_t = e^{-r(T-t)}\mathbb{E}_t^Q\left[\max\{V_T - (1+c)D, 0\}\mathbf{1}_{\{t^*>T\}}\right], \tag{4}$$

where the indicator function means that the insurer remains solvent at maturity. In addition, the stock price of the insurer S_t can be defined as $S_t = \frac{E_t}{\mathcal{N}}$, where $\mathcal{N}$ denotes the number of shares outstanding.

A contingent equity facility grants the insurer the right to issue $\mathcal{N}_{\text{new}}$ new shares when both the accumulated catastrophe losses and the stock price reach their trigger conditions. At that time, its asset value increases due to the timely fund injection from the issuance of new shares, and the post-exercise asset value V_t' becomes $V_t + \mathcal{N}_{\text{new}} \times K$. However, the new shares also dilute the percentage ownership of existing equity holders since the number of shares outstanding increases to $\mathcal{N}' = \mathcal{N} + \mathcal{N}_{\text{new}}$. Similar to Lo et al. (2013) and Sundaresan and Wang (2015), the post-exercise stock price can be expressed as $S_t' = \frac{E_t'}{\mathcal{N}'}$, where the post-exercise equity value E_t' is expressed as

$$E_t' = e^{-r(T-t)}\mathbb{E}_t^Q\left[\max\{V_T' - (1+c)D, 0\}\mathbf{1}_{\{t^*>T\}}\right], \quad t \geq t^{\#}, \tag{5}$$

and the exercise time $t^{\#}$ is the first passage time of the contract trigger:

$$t^{\#} = \inf\{t \in \mathcal{T} \mid \sum_{i=0}^{N_t} L_i \geq L \wedge S_t' \leq K\}. \tag{6}$$

To the best of our knowledge, the net-worth provisions have received limited attention in the literature. Recall that when the insurer exercises a contingent equity, the seller is obligated to purchase the new shares issued by the insurer. However, in case of severe catastrophes that significantly deteriorate the insurer's financial status, funds thus injected by the seller may not save the insurer from insolvency. To avoid such worthless fund injections, the net-worth provision requires the insurer to have a certain level of net assets when exercising contingent equity facilities. Thus, we model the net-worth provision to knock out the option when the post-exercise asset value still falls below the default boundary. Interestingly, our later quantitative analysis shows that the absence of this provision may not simultaneously improve the benefits of sellers and buyers, making the contract non-tradable.

3 Valuation of Contingent Capital

Section 3.1 develops the TTMJ, a sophisticated tree model to model the corresponding capital structure changes and firm value injections for the valuation of a complex contingent capital—such as the contingent equity facility discussed in this paper—that cannot be solved analytically. In addition, the valuation of contingent equity differs from the perspectives of the seller and the buyer since the seller's fund injections do not match the buyer's gain. Since rational participants do not trade in such a way as to sacrifice their benefits, we assess the tradability of contingent equity in **Section 3.2** by comparing the seller's minimum with the buyer's maximum acceptable prices. Finally, we revise the trade-off theory in **Section 3.3** to consider the scenario in which the insurer's assets do not cover catastrophe losses and analyze how the benefits of and losses from buying contingent equity are distributed to claimholders and their behaviors.

3.1 Valuation Method: TTMJ

To evaluate the changes in the values of claimholders due to the issuance of contingent equity and the value of potential capital injections, we build a TTMJ by combining the forest structure proposed in Liu et al. (2016) and the lattice (or tree) structure proposed in Wang and Dai (2018). Generally speaking, a tree structure divides a time period, say, the life period of a contingent equity facility, into discrete time steps and discretely specifies the evolution of an underlying asset, say, the firm value process defined in Equation (2). Our TTMJ is composed of two trees that model the evolutions of the firm value before or after the exercise of contingent equity.

3.1.1 Review of Liu et al. (2016) and Wang and Dai (2018)

The twin tree structure and its connection for simulating firm value injections due to the exercise of contingent equity can be modeled by mimicking the forest structure proposed by Liu et al. (2016) as shown in Figure 1.[17] Specifically, it simulates financing early redemption of callable bonds with the firm value and corresponding change of the debt structure with the transition from the upper tree to the lower one. The downward jump of the call price CP in the firm value due to bond redemption reduces the firm value from node U to node W (in the upper tree). Liu et al. use the trinomial structure proposed by Dai et al. (2010) to build the outgoing branches from node W to nodes X, Y, and Z to model the evolution of after-redemption firm value.

To simulate drops in the firm value due to catastrophic events, we take advantage of the lattice structure proposed by Wang and Dai (2018), which decomposes a time step into diffusion and jump phases, as illustrated in Figure 2.[18] The diffusion phase models the log-normal diffusion of the firm value as denoted by the first term on the right-hand side of Equation (2). The jump phase simulates

[17] See Figure 3 in Liu et al. (2016).
[18] See Figure 2 in Wang and Dai (2018).

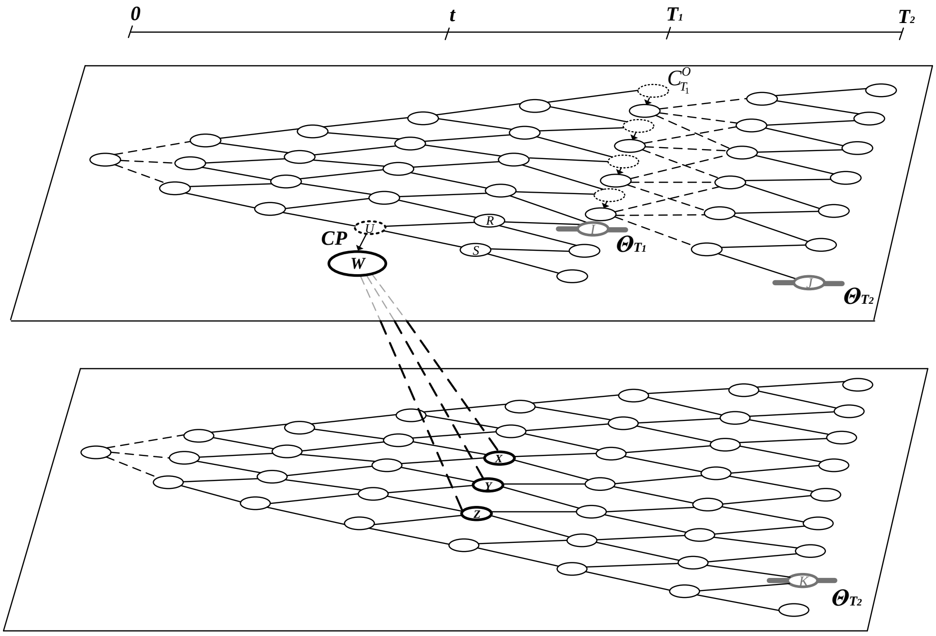

Figure 1: Forest Model. We illustrate the forest structure proposed in Figure 3 of Liu et al. (2016) composed of two trees for simulating the firm value process before and after redeeming a callable bond. The change in the firm value and the capital structure due to bond redemption at time t is modeled by transitions from the upper tree to the lower tree.

the firm value jump magnitude due to catastrophe losses modeled by revising the lattice structure for log-normal jumps in Wang and Dai (2018).[19] As shown in Figure 2, Wang and Dai (2018) simplify the jump magnitudes by discrete upward and downward jumps with integers +1 and −1 to reflect the jump size h and $-h$,[20] respectively. The accumulated jump sizes can be modeled as h times the sums of integers, as illustrated by the numbers in red beside that node. For example, the upper (lower) blue curve denotes the scenario in which the loss at the first time step is $-h$ and that at the second time step is 0 ($-h$); the accumulated loss at the end of the second time step is $-h$ ($-2h$). Additional states for each tree node that reflect different accumulated losses are inserted into the tree, and the contract values under different accumulated losses are evaluated by standard backward induction, as described in Wang and Dai (2018).

3.1.2 Construction of `TTMJ`

The values of contingent equity and claimholders, such as the equity value, debt value, tax benefit, etc., can be evaluated by applying backward induction with inserted states, as proposed in Liu et al. (2022), on a carefully designed `TTMJ`, as illustrated in Figure 3. Borrowing the forest concept proposed in Liu et al. (2016), our two-time step `TTMJ` is composed of lower and upper trees for simulating the firm value processes before and after exercising contingent equity facilities, respectively. In addition,

[19] Unlike Wang and Dai (2018), we model the loss magnitude by a zeta distribution, as described in Equation (1).
[20] The jump size h in this paper is set to $m\times$ 10 billion, as stated in Section 2.

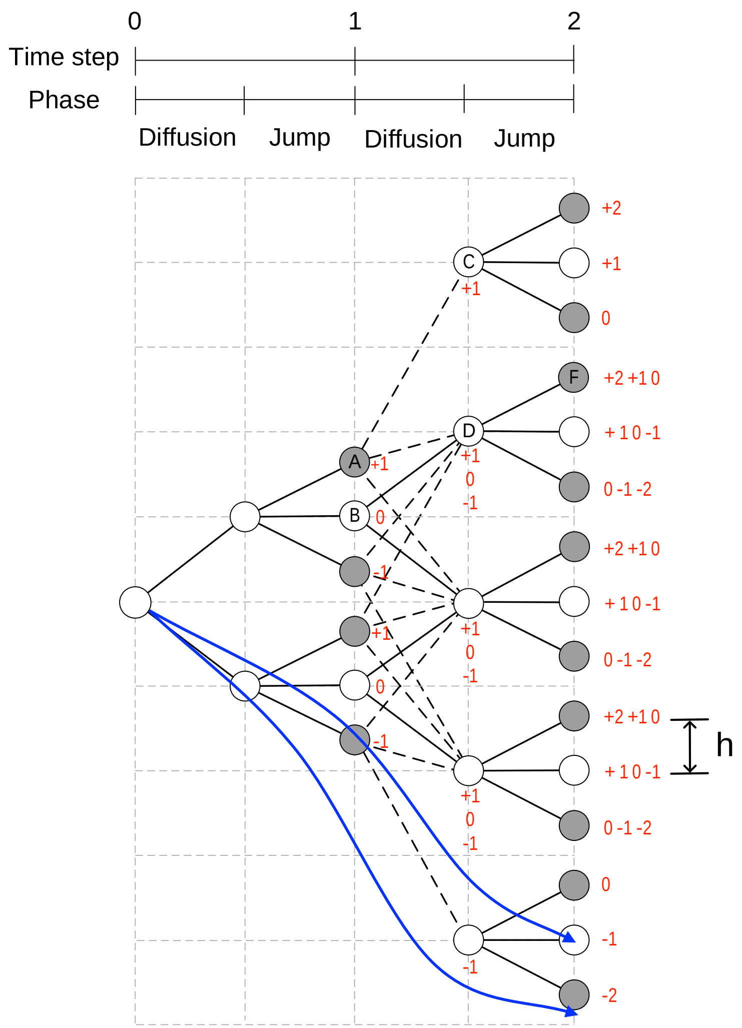

Figure 2: Tree with States for Recognizing Value Accumulations. We illustrate the tree structure for pricing a contingent equity facility proposed in Figure 2 of Wang and Dai (2018). Each time step is decomposed into a diffusion phase and a jump phase to model the jump-diffusion stock price, as described in Equations (1)–(4) of Wang and Dai (2018). Gray circles discretely model the log-normal jump magnitudes defined as a basic loss unit h times an integer. Each red integer i beside a node reflects an inserted state (of that node) representing the scenario in which the accumulated jump is ih. The blue curves denote two different asset value paths with accumulated jumps $-h$ and $-2h$, respectively.

the insurer defaults when its asset value falls below the default boundary (the red dashed curves) defined in Equation (3) for the first time. To avoid unstable pricing results due to the nonlinearity error problem defined in Figlewski and Gao (1999), we follow Liu et al. (2016) by making gray nodes coincide with the default boundary.[21] Next, the nodes positioned above the gray nodes are equidistant and situated[22] in accordance with Cox et al. (1979)'s tree structure.

To model the log-normal diffusion and the occasional catastrophe loss components of the firm value,

[21]This is analog to placing nodes I (J and K) that coincide with the default boundary Θ_{T_1} (Θ_{T_2}), as illustrated in Figure 1.

[22]This setting is similar to the white nodes on the mesh as depicted by the grids in Figure 2.

as stated in Equation (2), we mimic Wang and Dai (2018)'s design by decomposing a time step into diffusion and jump phases and using different branch structures to simulate diffusion and jumps, as illustrated in Figure 2. In Figure 3, the trinomial structure during the diffusion phase—for instance, the outgoing branches from node V_0 to A_1, B_1, and C_1, from $a_{1,1}$ to B_2, C_2, and D_2, or from $a''_{1,\ell}$ to B'_2, C'_2, and D'_2—is constructed by the method proposed in Appendix A of Dai et al. (2010).[23] The branches and corresponding probabilities during the jump phase, such as the outgoing branches from A_1 to $A_{1,0}$, $A_{1,1}$, ..., $A_{1,\ell}$ that reflect 1, 2, ... ℓ units of catastrophe losses, respectively, are determined in Section 2.1. The drop in firm value $(1-\tau)cD\Delta t$ due to coupon payments is simulated by inserting an after-coupon firm value node (e.g., the purple node $a_{1,1}$) right after the before-coupon node (e.g., the node $A_{1,1}$). The trinomial branches from purple nodes, such as the branches from $a_{1,1}$ to B_2, C_2, and D_2, are constructed by Dai et al. (2010)'s method.

Note that contingent equity facilities are exercised when the accumulated losses reach L (or ℓ unit losses) and the post-exercise stock price is less than the strike price K, as stated in Equation (6). These exercises result in fund injections to the firm value and changes in the firm's capital structure reflected in red or cyan branches for modeling the transition from the lower tree to the upper one.[24] To model the changes in accumulated losses, we follow Wang and Dai (2018) by inserting states into each node (e.g., the red integers in Figure 2) to reflect corresponding accumulated losses. For simplicity, in Figure 3, we only list the states ℓ (ℓ and $\ell-1$) right behind the node $a_{1,\ell}$ ($c_{2,\ell-1}$) and explain how to calculate accumulated losses. In node $a_{1,L}$, the loss at time step 1 and hence the accumulated loss reaches ℓ units, so the option can be exercised (reflected by the red branch jumping to $a''_{1,\ell}$) when the prevailing stock price S_t is lower than K. The corresponding changes in the capital structure and claimholders' values are analyzed in Section 2.4. In node $c_{2,\ell-1}$, the accumulated losses depend on the evolution path and determine whether the option can or cannot be exercised. Taking the cyan path as an example, the loss at time step 1 (2) is 1 ($\ell-1$) unit, and the accumulated loss at node $c_{2,\ell-1}$ is $1+(\ell-1)=\ell$ units. The outgoing transition branch to the upper tree reflects the fact that the option can be exercised given the post-exercise stock price is lower than K. The green evolution path, in turn, denotes that the loss at time step 1 is 0 units, and thus the accumulated loss in $c_{2,\ell-1}$ is $0+(\ell-1)=\ell-1$ units. The outgoing branches connect to the states in the lower tree to reflect the fact that the option cannot be exercised. Note that applying backward induction for different successor states yields different contingent claim values that reflect the impact of different accumulated losses.

[23]Their mean-tracking method constructs feasible trinomial branches from a node to three selected adjacent nodes in the next time step given the layout of nodes following Cox et al. (1979)'s tree structure.

[24]This is analog to the tree transition for modeling financing callable bond redemption with firm value, as illustrated in Figure 1.

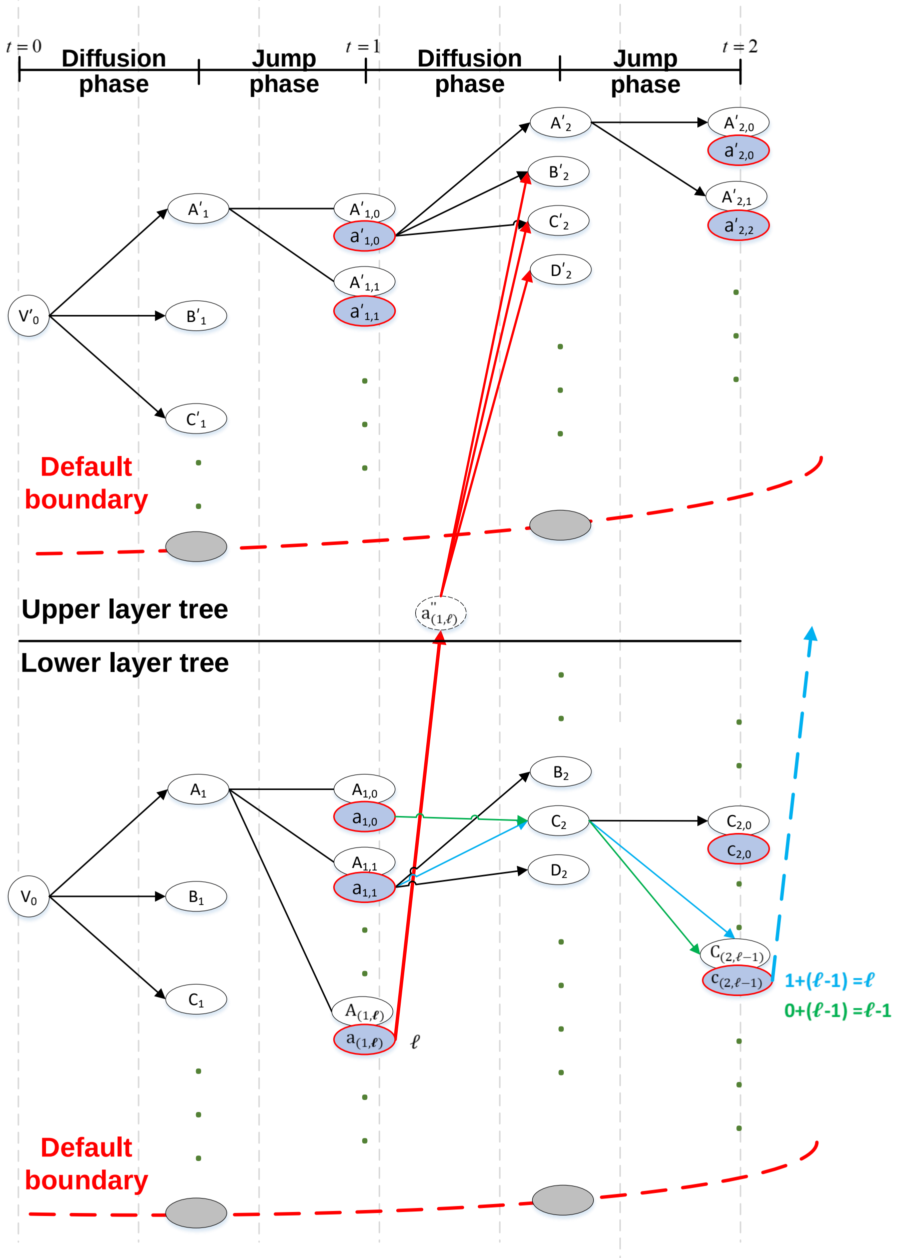

Figure 3: A Two-Time-Step TTMJ. A `TTMJ` comprises the upper- and the lower-layer trees that model the evolution of the asset value process after and before exercising contingent equity facilities, respectively. Each time step can be decomposed into diffusion and jump phases, as illustrated in the upper corner. Red-dashed curves denote the default boundaries B_t. Nodes denoted by the capital letter and the lowercase one represent the asset values before and after paying coupons, respectively. The first and the second subscripts for each node symbol represent the time step and the units of loss at that time step, respectively.

3.2 Minimum/Maximum Acceptable Prices

The valuation of a complex contingent capital, such as a contingent equity facility in our scenario, involves an endogenous problem because the contract value depends on the insurer's stock price, which in turn is influenced by the purchase price and the post-purchase firm value. This endogeneity is also noted by Lo et al. (2013), Sundaresan and Wang (2015), and Glasserman and Nouri (2016). In this section, we develop an iterative valuation method that repeatedly applies our `TTMJ` to address this endogeneity, and we analyze the minimum required price and the maximum acceptable price of a CatEPut from the seller's and the buyer's perspectives in **Sections 3.2.1** and **3.2.2**, respectively.

3.2.1 Seller's Perspective

As a seller of contingent equity, the willingness to bear catastrophe risk depends on whether the deal price (`Deal`) is higher than its issuing cost (`IC`). Without sacrificing the sellers' benefits, the seller requires a price above or equal to the `IC` to replicate the potential compensation payments minus the value of the obtained stock shares. Note that the `IC` can be valued as the expected discounted payoffs of the contingent equity under the risk-neutral measure as introduced in **Section 3.1** as $\texttt{IC}_t = e^{-rt^{\#}}\mathbb{E}_t^Q\left[\mathcal{N}_{\text{new}} \times (K - S'_{t^{\#}})\right]$.

However, once the insurer purchases the contingent equity at the deal price, the cash outlay for the contract decreases the insurer's asset value (from V_0 to $V_0 - \texttt{Deal}$), making the expected share price less valuable. As a result, the seller demands a higher premium, which in turn leads to another decrease in the expected share price. Lo et al. (2013) points out that this endogenous problem can have a significant impact on the valuation of contingent capital, especially when the number of new shares available for issuance, $\mathcal{N}_{\text{new}}$, is high. To solve this endogenous problem between `IC` and `Deal`, we repeatedly calculate the `IC` by tuning the after-purchase firm value (i.e., $V_0 - \texttt{Deal}$) until the two values converge and the relative error is less than 10^{-8}.

Figure 4 shows the flowchart used to value the contingent equity from the seller's perspective. First, given the model parameters, we calculate `IC` without considering the endogenous problem. Then, we set `Deal` to the `IC` and update the firm value of the insurer to be $V_0 - \texttt{Deal}$ to recalculate the `IC`. This step is repeated until the `IC` converges to `Deal`. The resulting equilibrium price is the minimum price required by the seller to sell the contingent equity; for simplicity, we term this the *ask price*.

3.2.2 Buyer's Perspective

As a buyer of contingent equity, the willingness to pay the purchase price `Deal` depends on the potential benefit of receiving emergency capital injections. To weigh the pros and cons of purchasing a contingent equity facility, we analyze the benefits from the perspective of a firm value maximizer and an equity value maximizer, respectively. If a contract purchase decreases the insurer's firm value (or equity value), the insurer has no incentive to purchase it. Therefore, the maximum price acceptable to the buyer, or the bid price, is the highest price that does not reduce the firm-levered value (or equity value).

Define the pre-purchase firm-levered value as $V_0^L \equiv E_0 + D_0$, and the post-purchase firm-levered value as $V_0'^L \equiv E_0' + D_0'$. The equity value E_0 and debt value D_0 prior to the purchase of contingent equity are defined in (4) and (3), and can be calculated by applying backward induction on the `TTMJ` with firm value V_0. Likewise, the corresponding post-purchase values E_0' and D_0' are calculated with

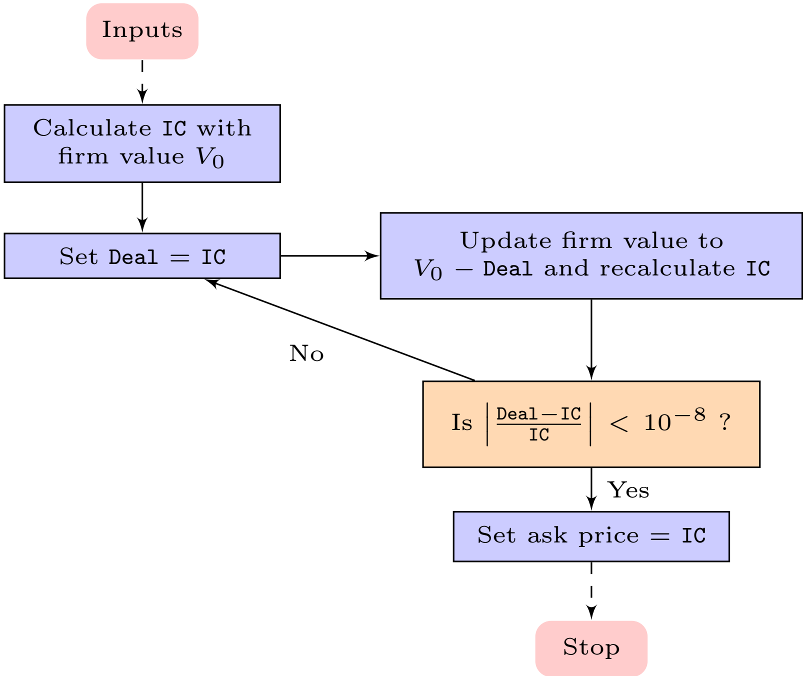

Figure 4: Valuation from Seller's Perspective

the firm value $V_0 - \texttt{Deal}$. Finally, the valuation of the bid price is equivalent to finding a `Deal` that makes $V_0'^L = V_0^L$ (or $E_0' = E_0$).

The existence of the bid price can be proved by the intermediate value theorem. For the firm value maximizer, it is obvious that $V_0'^L > V_0^L$ when $\texttt{Deal} = 0$, since the contingent equity provides the buyer with the possibility of receiving cash inflow without incurring any costs. However, $V_0'^L < V_0^L$ when $\texttt{Deal} = V_0$, because the cash outlay for the contract reduces the buyer's asset value to hit the default boundary. Therefore, there must be a `Deal` in the interval $(0, V_0)$ that meets the condition $V_0'^L = V_0^L$. Similar arguments can be applied to the equity value maximizer, except that the loss of equity value due to dilution of new shares from the exercise of contingent equity may outweigh the benefits of fund injections. Under very extreme scenarios, E_0' might be smaller than E_0 even when $\texttt{Deal} = 0$ and the bid price is a negative number.

Figure 5 shows the flowchart used to value the contingent equity from the buyer's perspective. First, given the model parameters, we set the initial purchase price `Deal` equal to the ask price determined in **Section 3.2.1**. Next, we determine that the objective function is a firm value maximizer or an equity value maximizer. We calculate $V_0'^L$ and E_0' to examine how the contract purchase changes the insurer's firm-levered value or equity value. Similar to the valuation of ask prices, evaluating bid prices also leads to price endogeneity. Increasing `Deal` reduces the after-purchase firm value $V_0 - \texttt{Deal}$ and hence decreases $V_0'^L$ (or E_0'), which influences the pricing result to meet the termination condition $V_0'^L \approx V_0^L$ (or $E_0' \approx E_0$). Thus, we repeatedly adjust the deal price `Deal` with digit-by-digit calculation to evaluate the firm's levered value (or the equity value) with our `TTMJ` until the termination condition is met.

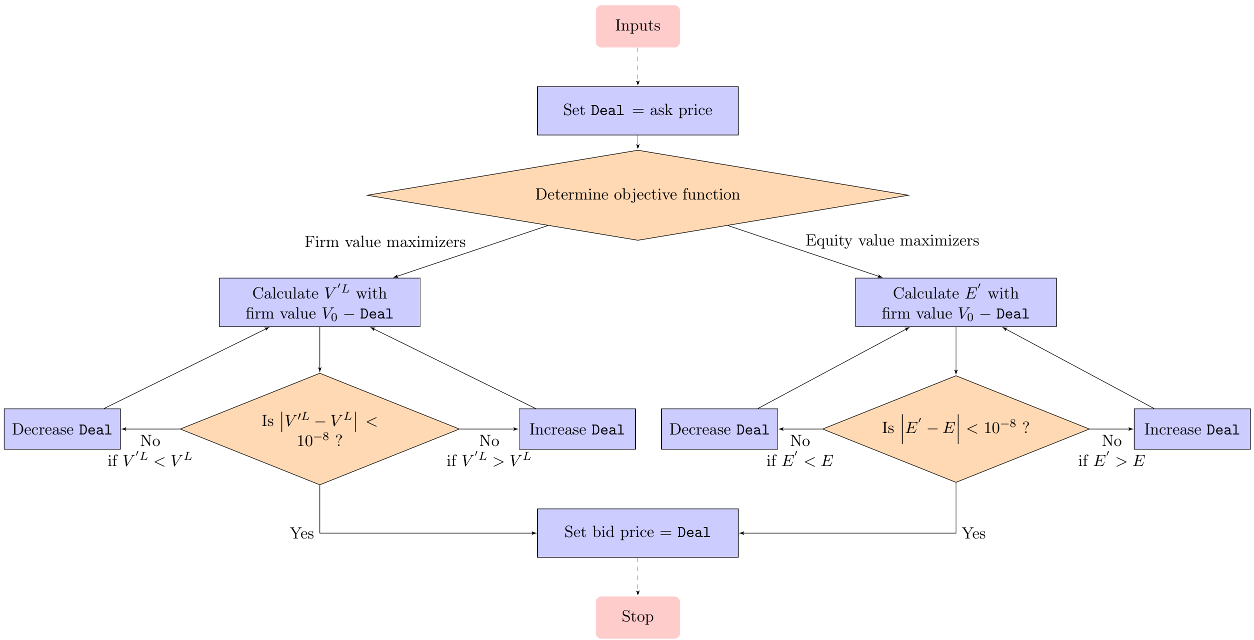

Figure 5: Valuation from Buyer's Perspective

After deriving the ask and bid price for a contingent equity facility, as in Figures 4 and 5, respectively, we determine whether the contract can be dealt with after negotiations. Since the seller (buyer) does not accept a deal price that is lower (higher) than the ask (bid) price, contingent equity can be traded only when the bid price is higher than the ask price (or the feasible trading region exists). This property can be used to analyze whether a contingent equity facility is tradable by comparing various provisions mentioned in the relevant literature or real-world contracts in our subsequent analysis.

3.3 Revised Trade-off Theory

Trade-off theory (see Kraus and Litzenberger, 1973) analyzes how the levered value of a firm, such as an insurer in our analysis, is affected by its leverage ratio (or total liabilities). Specifically, the levered firm value (V_t^L) at time t, defined as the sum of values of claimholders like shareholders (E_t) and debtholders (D_t), can be expressed as a function of the leverage ratio $\eta \equiv D/V_0$ and decomposed into the firm's asset value V_t modeled in Equation (2) plus the tax benefits $\mathtt{TB}_t$ minus the bankruptcy costs $\mathtt{BC}_t$:

$$V_t^L(\eta) \equiv E_t(\eta) + D_t(\eta) = V_t + \mathtt{TB}_t(\eta) - \mathtt{BC}_t(\eta). \tag{7}$$

However, Lakdawalla and Zanjani (2012) indicate that the insured amount might exceed the insurer's assets, a situation that traditional trade-off theory fails to account for and cannot explain. This implies that catastrophe losses (the rightmost term in Equation (2)) could exceed the firm's prevailing asset value and cause $V_{t+\Delta t}$ to be negative.[25] This scenario implies that either the insured does not receive full compensation or that other institutions, such as insurance guarantee funds, should cover

[25] That might be why the government may enact insurance guarantee funds to compensate policyholders for losses caused by insurance company insolvencies, as stated in Cummins (1988).

the shortfall. To revise the problem, we incorporate loss compensation ($\texttt{LC}$) to reflect the present value of the insured loss or shortfall as

$$V_t^L(\eta) \equiv E_t(\eta) + D_t(\eta) = V_t + \texttt{TB}_t(\eta) - \texttt{BC}_t(\eta) + \texttt{LC}_t(\eta). \tag{8}$$

Note that if the catastrophe loss is always less than the asset value, then the $\texttt{LC}$ is zero.

Next, we explore the impact of the purchase of contingent equity on our revised trade-off theory. Insurers pay the purchase price $\texttt{Deal}$ (financed by the insurer's asset) to obtain potential emergency capital injections, but this action also leads to equity dilutions. To evaluate the rationale behind the contract purchase, we assess the changes in the firm-levered value (ΔV_t^L), the equity value (ΔE_t), and the debt value (ΔD_t) due to the purchase, respectively. Following the revised trade-off theory (8), we analyze the impact of contract purchases as

$$\Delta V_t^L(\eta) \equiv \Delta E_t(\eta) + \Delta D_t(\eta) = \Delta V_t(\eta) + \Delta\texttt{TB}_t(\eta) - \Delta\texttt{BC}_t(\eta) + \Delta\texttt{LC}_t(\eta), \tag{9}$$

where $\Delta V_t(\eta)$, $\Delta\texttt{TB}_t$, $\Delta\texttt{BC}_t$, and $\Delta\texttt{LC}_t$ represent the changes in firm asset value, tax benefit, bankruptcy cost, and loss compensation resulting from the purchase of the contingent equity, respectively. Note that the change in the firm asset value is dependent on η, which indicates the discrepancy between the price paid by the buyer ($\texttt{Deal}$) and the value of the potential capital injection received by the buyer ($\texttt{IC}_t$). That is, $\Delta V_t(\eta) = \texttt{IC}_t(\eta) - \texttt{Deal}(\eta)$. When the purchase price $\texttt{Deal}$ precisely matches the seller's minimum required price $\texttt{IC}_t$, the firm asset value does not change due to the purchase. However, if $\texttt{Deal}$ is set higher through negotiation between the contract buyer and seller, increasing $\texttt{Deal}$ decreases the firm asset value and negatively impacts the overall firm-levered value. Detailed discussions of the changes in claimholders' values are given in **Section 4.4**.

4 Quantitative Analyses

This section quantitatively analyzes the tradability of various types of contingent equity facilities and the benefits for insurers. **Section 4.1** describes how we determine the base case by retrieving parameters from real-world data such as U.S. catastrophe loss data, RLI financial reports, and the `CatEPut` contract signed by RLI. **Section 4.2** calculates the minimum acceptable price from the seller's perspective, or the ask price, by the method proposed in **Section 3.2.1**. **Section 4.3** first calculates the maximum acceptable price from the buyer's perspective, or the bid price, by the method proposed in **Section 3.2.2**, and then explores the feasible trading region to find the "tradeable" contract design. **Section 4.4** analyzes the change in the firm-levered value ΔV_t^L and equity value ΔE_t due to the purchase of different types of `CatEPuts` to find the appropriate contract provisions for insurers. Finally, we examine the robustness of our model and analyses in **Section 4.5**.

4.1 Parameter Settings

The first `CatEPut` contract was signed between RLI Corp and Centre Re in October 1996,[26] giving RLI the right to sell up to 50 million shares to Centre Re right after the event in which the losses of California earthquakes depleted its reinsurance program, provided RLI could continue to operate after capital injections. To reproduce the transaction scenario at that time and analyze the decisions for both parties, we extracted the financial status of RLI and the `CatEPut` provisions mainly from the RLI's 10-K reports[27] to determine model parameters.

To estimate the catastrophe parameters, we downloaded the total damages caused by earthquakes in the United States from 1902 to 2022 from the EM-DAT database. Figure 6 shows the annual counts of earthquakes recorded in the EM-DAT database. The results show a maximum of three observations per year, which is consistent with the principle that catastrophes are rare events with no clear trend or pattern. With this in mind, our empirical estimates exclude earlier data and select a sample period from 1950 to 1995. Figure 7 shows the distribution of total damages incurred from the 28 earthquakes during the sample period. This distribution aligns with the zeta distribution and supports our model assumption.

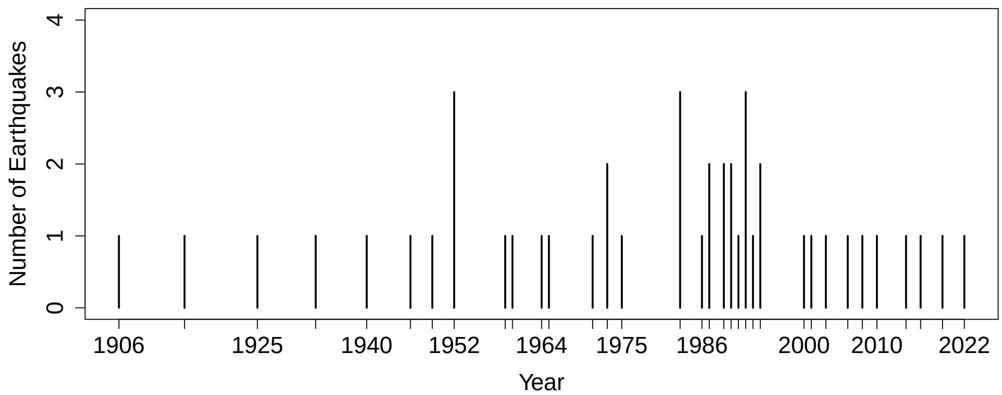

Figure 6: Annual Counts of Earthquakes in the U.S.
The sample period, spanning from 1902 to 2022, was obtained from the EM-DAT database.

Table 1 summarizes the parameters into four categories, the first of which describes the parameters associated with catastrophes. We calibrate the intensity parameter λ by fitting a Poisson proces to the annual observations of earthquakes and calibrate the magnitude of catastrophe losses by fitting Equation (1) to the total damages of each earthquake in our sample. We employ the maximum log-likelihood method and obtain an estimated jump intensity λ of 0.61 and an estimated shape parameter s of 1.09. Both parameters are assumed to be the same under both physical and risk-neutral measures, which implies that catastrophe risks are not diversifiable, as suggested by Merton (1976) and Jaimungal

[26] We denote the contract effective date as time 0 in latter discussions.
[27] `https://investors.rlicorp.com/sec-filings/default.aspx`

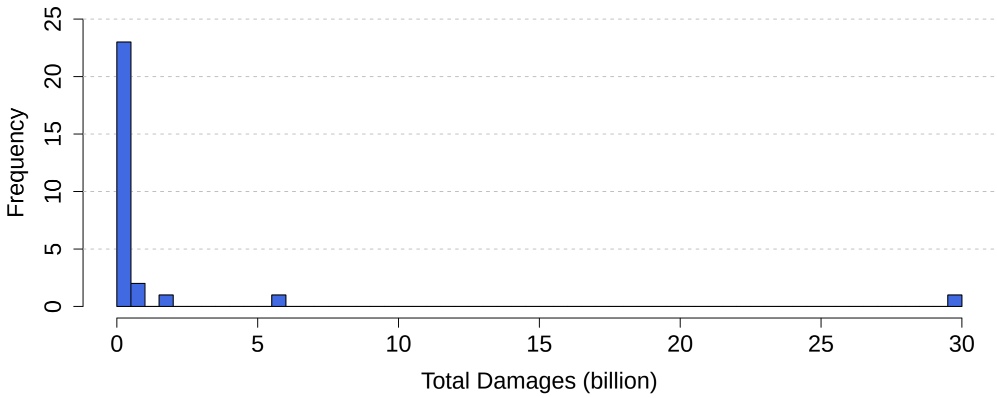

Figure 7: Distribution of Earthquake Losses in the U.S.
The sample period spans from 1950 to 1995 and was obtained from the EM-DAT database.

and Wang (2006).

The second category of parameters in Table 1 describes the financial status and capital structure of RLI. The initial value of the firm assets ($V_0 = \$905,764,000$), used in Equation (2) to simulate the evolution of firm value, is estimated as the total liabilities plus the market value of equity, following the approach outlined in Eom et al. (2004). The volatility of asset value ($\sigma = 5\%$) follows the setting in Duan and Yu (2005), Lo et al. (2013), and Himmelberg and Tsyplakov (2020). The total liabilities ($D = \$645,400,000$), the number of shares outstanding ($\mathcal{N} = 7,800,000$), and the effective tax rate ($\tau = 27.1\%$) are retrieved from 10-K reports. Thus, the leverage ratio of RLI, $\eta \equiv D/V_0$, is approximately 71%. To cover the `CatEPut` price discussion for companies with different leverage ratios, we will fix the asset value V_0 and use multiple leverage parameters $\eta \in \{0.1, 0.2, \ldots, 0.9\}$ to perform the analysis. The coupon rate ($c = 6\%$) is estimated as the average rate of RLI's total liabilities extracted from the Mergent FISD database. The market share ($m = 0.18\%$) of RLI is retrieved from the California Department of Insurance.[28]

The third category in Table 1 focuses on the parameters associated with the `CatEPut` contract extracted from the 10-K report. RLI's `CatEPut` is a three-year option ($T = 3$) that allows the insurer to issue 50,000 new shares (i.e., $\mathcal{N}_{\text{new}} = 50,000$) if its post-exercise stock price is lower than 1000 (i.e., $K = 1000$) and the accumulated catastrophe losses are higher than \$216 million (i.e., $L = \$216,000,000$). Note that the market price of RLI shares was only 33.38 at the time, significantly lower than the strike price. This implies that the contract could almost be considered a single trigger contract driven by catastrophe losses. Since contingent capital is widely discussed in the literature as featuring a dual-trigger mechanism (Lo et al., 2013; Sundaresan and Wang, 2015), we further consider a hypothetical at-the-money scenario in subsequent analyses to ensure that our conclusions are not affected. Likewise, the catastrophe trigger level of this contract is about 24% of RLI's asset value,

[28] `http://www.insurance.ca.gov/01-consumers/120-company/04-mrktshare/`

Table 1: Parameters for Pricing RLI's `CatEPut`

Category	Symbol	Definition	Value
Catastrophe parameters	λ	Intensity of Poisson process	0.61
	s	Shape parameter of Zeta distribution	1.09
RLI Corp parameters	V_0	Insurer's assets value at time 0	$905,764,000$
	σ	Volatility of insurer's asset (%)	5
	D	Total liabilities	$645,400,000$
	$\mathcal{N}$	Shares outstanding	$7,800,000$
	τ	Effective tax rate (%)	27.1
	c	Insurer's coupon rate (%)	6
	m	Insurer's market share (%)	0.18
`CatEPut` parameters	T	Time to maturity (years)	3
	$\mathcal{N}_{\text{new}}$	New shares issued after exercising `CatEPut`	$50,000$
	K	Strike price	1,000
	L	Trigger level of catastrophe losses	$216,000,000$
Environmental parameters	α	Liquidation cost (%)	40
	r	Risk-free rate (%)	5

and our analysis will be performed by setting $L/V_0 \in \{0.1, \ldots, 0.5\}$ to explore the potential differences caused by the catastrophe trigger condition.

Finally, the last category in Table 1 includes other environmental parameters. The liquidation cost α is set to 40%, aligning with the range of 10% to 50% commonly used in the literature, including work by Sundaresan and Wang (2015) and Chen et al. (2017). The risk-free rate r is set to 5% as in Cox et al. (2004) and Himmelberg and Tsyplakov (2020).

4.2 `CatEPut` Ask Price

Here, we delve into the valuation of `CatEPut`, taking into consideration various contract provisions, corporate leverage characteristics, and catastrophe trigger thresholds. Our analysis begins by calculating the ask price of a European-style `CatEPut`, with a specific focus on the absence of the net-worth provision, as this is a commonly discussed standard contract in the literature (Cox et al., 2004; Jaimungal and Wang, 2006). To further enhance our understanding, we extend our investigation to explore the impact of an early exercise provision in **Section 4.2.1** and a net-worth provision in **Section 4.2.2**. To ensure the applicability of our findings beyond specific cases like RLI, which involve extremely deep in-of-the-money options, we replicate our analyses in **Section 4.2.3** for hypothetical contracts with a strike price equal to the stock price right before `CatEPut` issuance. By conducting these comprehensive analyses, we seek to explore the rationality of the provision design, providing important considerations for market participants in this domain.

4.2.1 Early Exercise Provision

In Panel A of Table 2, we evaluate the ask price of a European-style CatEPut without the net-worth provision under various ratios of catastrophe loss trigger levels to the initial firm value (L/V_0), presented in the first column, and the leverage ratio (η), indicated in the first row. To account for the impact of the leverage ratio on equity value, we list the corresponding stock price before the issuance of CatEPut (S_0) in the second row. We highlight in red the ask prices for four scenarios that sandwich the RLI case stated in Table 1, where a leverage ratio is 71% and a trigger level of catastrophe losses is set at 24% of the asset value.

First, we sketch the pattern of the ask price of CatEPut contracts. It decreases as the trigger level of catastrophe losses (L) increases. This decline is consistent with theoretical expectations due to the diminishing likelihood of exercising the CatEPut. However, the ask price does not follow a monotonic pattern with respect to the leverage ratio (η). Take $L/V_0 = 0.2$ as an example: the theoretical price rises with η initially, then decreases, with the maximum price at $\eta = 0.7$. Typically, increases in the leverage ratio lead to a higher intrinsic value of the CatEPut contract. However, high-leverage firms are associated with a higher probability of default compared to low-leverage firms, posing a higher likelihood that the firm defaults prior to its loss reaching the trigger level L. Since this increasing likelihood decreases the chance to exercise CatEPuts, the ask price of CatEPut contracts may decrease as the leverage ratio rises, particularly in scenarios with high leverage levels. Notably, most previous studies (Cox et al., 2004; Jaimungal and Wang, 2006; Lo et al., 2013) do not consider the potential impact of the buyer's bankruptcy within the valuation of CatEPut, limiting their ability to establish a comprehensive understanding beyond the monotonic relationship with the stock price.

In terms of the magnitude of CatEPut prices, the ask price for the RLI case falls within the range of \$4,362,869 and \$12,472,690, representing approximately 0.48% to 1.38% of the asset value, respectively. Taking a broader view across Table 2, the highest ask price reaches \$20,874,974, equivalent to 2.31% of the asset value. The costs of these contracts demonstrate significant affordability, aligning harmoniously with the essence of insurance contracts and supporting the validity of the pricing mechanism.

Next, we proceed to calculate the ask prices of American-style CatEPut contracts and present the corresponding early exercise premium in Panel B of Table 2. The early exercise premium is defined as the incremental change relative to the corresponding European-style contracts. Note that the early exercise premium is not necessarily positive. Following the assumption made by Lo et al. (2013), we consider that the buyer exercises the contract immediately when the dual-trigger conditions are met to obtain emergency capital injection. This buyer's early exercise policy acts as a barrier option and may not be the optimal exercise decision (i.e., the exercise value might be lower than the continuation value to keep CatEPut unexercised). Consequently, the early exercise premium may be theoretically negative.

Table 2: Ask Prices of European-style and American-style CatEPuts

This table shows the ask price under various catastrophe trigger levels (L) and leverage ratios (η). S_0 is the stock price before the issuance of CatEPuts. Panel A shows the price for a European-style contract without the net-worth provision, and Panel B illustrates the early exercise premium, defined as the incremental change in American-style contracts relative to their European-style counterparts. The four scenarios that sandwich the RLI case stated in Table 1 are highlighted in red.

L/V_0 \ η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
S_0	104.72	93.31	81.91	70.51	59.14	47.83	36.68	25.79	15.73
Panel A: Ask Price of European-Style CatEPut									
0.1	18,922,247	19,203,061	19,483,851	19,764,550	20,045,124	20,324,975	20,604,487	20,874,974	19,767,626
0.2	11,466,222	11,634,615	11,802,986	11,971,276	12,139,446	12,307,006	12,472,690	12,213,739	9,347,854
0.3	5,773,637	5,857,404	5,941,152	6,024,824	6,108,381	6,191,175	6,100,079	4,362,869	2,121,184
0.4	2,720,982	2,759,965	2,798,928	2,837,819	2,876,581	2,895,295	2,492,673	1,317,619	376,741
0.5	1,221,113	1,238,393	1,255,653	1,272,841	1,285,138	1,156,490	759,736	266,080	45,795
Panel B: Early Exercise Premium									
0.1	8.98%	8.85%	8.72%	8.59%	8.47%	8.34%	8.18%	7.98%	7.58%
0.2	7.77%	7.65%	7.54%	7.44%	7.32%	7.20%	7.05%	6.75%	6.87%
0.3	5.91%	5.83%	5.74%	5.66%	5.57%	5.45%	5.23%	4.76%	4.64%
0.4	5.12%	5.04%	4.97%	4.89%	4.80%	4.65%	4.49%	4.16%	3.47%
0.5	4.47%	4.40%	4.33%	4.25%	4.13%	3.95%	3.94%	3.34%	2.44%

Our findings show that the early exercise premium for CatEPut contracts ranges from 2.44% to 8.98%. Specifically, for the RLI example, the early exercise premium falls within the range of 4.76% and 7.05% (highlighted in red). The early exercise premium consistently increases with decrements in the trigger level of catastrophe losses, since the likelihood of CatEPut holders exercising their contracts early compared to a standard European-style option is also increased. These findings highlight the critical role of the early exercise provision in the valuation of CatEPut contracts under various catastrophe risk and leverage scenarios.

4.2.2 Net-Worth Provision

To prevent option sellers from injecting funds into financially unviable companies, RLI's CatEPut contract incorporates a net-worth provision. This provision prohibits the exercise of the CatEPut if the firm's value falls below a pre-agreed level, such as insolvency. Similar provisions can be found in other CatEPut contracts, including Horace Mann in 1997, LaSalle Re in 1997, and Trenwick in 2001. Although this provision is mentioned in Doherty (1997), we have not found similar provisions discussed extensively in more recent literature. Therefore, we will analyze how the presence of this provision affects the tradability of CatEPut contracts.

Table 3 provides the results analyzing the impact of the net-worth provision on CatEPut contracts. This impact is quantified by calculating the ratio of the ask price with the net-worth provision to that without the provision. A crucial distinction between the two prices arises when the contract is triggered but the buyer goes bankrupt. In such cases, the contract with the net-worth provision becomes worthless, whereas that without the provision still provides the predetermined funding. Consequently, the introduction of the net-worth provision leads to a decrease in the ask price. Panels A and B of

Table 3 illustrate the impact of this provision for European-style and American-style `CatEPut` contracts, respectively, offering quantitative insight into how the net-worth provision affects the pricing of `CatEPut` contracts under different exercise provisions.

The impairment of the net-worth provision follows a monotonically increasing pattern for both the catastrophe trigger level and the firm leverage ratio. For firms with low leverage and low probabilities of bankruptcy, the net-worth provision has minimal influence as the difference between contracts with and without the provision is negligible. However, for scenarios involving both high levels of the catastrophe trigger and firm leverage, the net-worth provision can render the `CatEPut` contract nearly worthless due to the elevated probability of bankruptcy and the difficulty in satisfying the trigger condition.

Table 3: Impact of Net-Worth Provision

This table shows the impact of the net-worth provision under various catastrophe trigger levels (L) and leverage ratios (η). This impact is quantified by calculating the ratio of the ask price with the net-worth provision to the price without the provision. Panels A and B show the impact for the value changes of European-style and American-style contracts, respectively. The four scenarios that sandwich the RLI case stated in Table 1 are highlighted in red.

L/V_0 \ η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Panel A: European-Style `CatEPut`									
0.1	99.97%	99.92%	99.73%	99.18%	97.60%	93.55%	83.82%	62.91%	35.23%
0.2	99.96%	99.87%	99.59%	98.76%	96.37%	90.19%	75.39%	45.94%	20.17%
0.3	99.93%	99.77%	99.25%	97.70%	93.27%	81.70%	57.23%	30.42%	11.92%
0.4	99.86%	99.53%	98.46%	95.30%	86.23%	63.46%	31.95%	12.12%	4.08%
0.5	99.69%	98.97%	96.64%	89.72%	70.44%	37.87%	12.63%	3.54%	1.03%
Panel B: American-Style `CatEPut`									
0.1	100.00%	100.00%	100.00%	100.00%	100.00%	99.96%	97.48%	80.31%	47.53%
0.2	100.00%	100.00%	100.00%	100.00%	99.98%	99.22%	92.03%	60.04%	26.07%
0.3	100.00%	100.00%	100.00%	99.99%	99.18%	92.98%	70.17%	38.12%	14.78%
0.4	100.00%	100.00%	100.00%	99.73%	95.54%	75.36%	39.02%	14.92%	4.86%
0.5	100.00%	100.00%	99.82%	97.18%	81.03%	45.12%	15.24%	4.30%	1.18%

Comparing the results between the two panels, we observe that the net-worth provision exerts a stronger influence on European-style contracts. In these contracts, early exercise is not allowed, meaning that if a catastrophic event significantly impacts the insurer and triggers the contract, it cannot provide an immediate capital infusion to decrease the buyer's probability of default. Consequently, the contract may end up being worthless. Conversely, American-style contracts allow for immediate capital infusion upon the trigger, thereby reducing the buyer's probability of default. This explains why the provision has a lower "discount" on the American-style `CatEPut`, which can be observed in the higher ratio in each scenario in Panel B than the corresponding ratio in Panel A. In the specific example of RLI, the presence of the net-worth provision exerts a significant influence on the price of `CatEPut` contracts, resulting in substantial discounts ranging from 38.12% to 92.03%. This wide range is determined by a combination of the `CatEPut` trigger probability and the probability of bankruptcy, underscoring the sensitivity of the net-worth provision's impact in this scenario (with L/V_0 ranging between 0.2 and 0.3, and η between 0.7 and 0.8).

In conclusion, although the early exercise premium, as discussed in Section 4.2.1, increases the value of `CatEPut` contracts, the net-worth provision has the opposite effect, reducing their value. However, in the RLI example, the impact of the net-worth provision appears to be more significant, overshadowing the mixed effect of the early exercise provision. This underscores the dominant role of the net-worth provision in shaping the overall value of `CatEPut` contracts.

4.2.3 Stock Price Trigger

Contingent capital contracts typically operate with dual triggers. However, RLI's high strike price transforms `CatEPuts` into a single trigger driven solely by catastrophe losses. To gain further insight, we analyze a hypothetical scenario in which the contracts have a strike price equal to the stock price prior to the `CatEPut` purchases. Note that the contracts are slightly out-of-the-money since the after-purchase stock price tends to be higher due to the endogenous relationship between future potential capital infusion and post-exercise stock price. These hypothetical contracts are referred to as "near-the-money contracts" in this study to distinguish them from real contracts. By examining this scenario, we can better understand the significance of the stock price trigger and its impact on the valuation of `CatEPut` contracts.

We analyze the `CatEPut` prices for near-the-money contracts in Table 4, which consists of three panels. Panels A and B of Table 4 replicate the analysis performed in Table 2, evaluating ask prices for European-style `CatEPuts` and early exercise premiums contributed by the corresponding American-style contracts, respectively, without the net-worth provision. Likewise, Panel C of Table 4 aligns with Panel B of Table 3, providing the discount resulting from the net-worth provision for American-style `CatEPut` contracts. This examination allows us to analyze the joint effects of the early exercise provision and the net-worth provision in the valuation.

The `CatEPut` prices in Panel A of Table 4 are significantly smaller than those in Table 2. Although the large numbers in Table 2 primarily represent the contracts' intrinsic values, the values in Table 4 reflect pure time values as the intrinsic values of these contracts are zero. The patterns in both tables are consistent, showing that the ask price decreases with the catastrophe trigger level L, and it initially increases with η and then decreases when η is high. However, the reason for the positive relationship between the ask price and η is different from the previous deep-in-the-money discussion. The previous analysis attributed the positive relation to the high intrinsic value of scenarios with high η. In contrast, the near-the-money contracts in Table 4 have no intrinsic value. Therefore, the main reason for the positive relationship is that a higher leverage ratio results in greater equity volatility, making the contract more valuable. This effect is particularly pronounced for high-vega options, such as the near-the-money contracts analyzed in this discussion.

In the RLI example, if the strike price is set to the prevailing stock price, the ask price of the

Table 4: Ask Prices of Near-the-Money CatEPuts

This table shows the ask price under various catastrophe trigger levels (L) and leverage ratios (η) for the hypothetical near-the-money contracts, where the strike price K is equal to the stock price before the CatEPut purchases. Panel A shows the price for a European-style contract without the net-worth provision. Panel B illustrates the early exercise premium, defined as the incremental change in American-style contracts relative to their European-style counterparts. Panel C shows the impact of the net-worth provision by calculating the ratio of the ask price with the net-worth provision to the price without the provision. The four scenarios that sandwich the RLI case stated in Table 1 are highlighted in red.

L/V_0 \ η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
K	104.72	93.31	81.91	70.51	59.14	47.83	36.68	25.79	15.73
Panel A: Ask Price of European-Style CatEPut									
0.1	143,292	159,325	177,472	196,550	216,010	235,547	251,166	260,015	225,725
0.2	139,719	154,408	170,540	187,097	203,537	218,780	229,010	222,160	131,947
0.3	120,581	130,663	141,095	151,313	160,748	167,667	162,959	98,535	31,900
0.4	87,455	92,712	97,906	102,697	106,460	106,396	82,520	32,979	5,864
0.5	53,293	55,635	57,856	59,659	60,086	50,304	27,183	6,826	719
Panel B: Early Exercise Premium									
0.1	55.76%	46.05%	37.06%	28.88%	22.11%	16.07%	11.02%	6.45%	4.70%
0.2	47.54%	39.68%	32.42%	25.98%	20.52%	15.65%	11.57%	8.19%	7.06%
0.3	27.05%	22.63%	18.61%	15.08%	12.01%	9.33%	7.18%	5.59%	4.91%
0.4	16.49%	13.90%	11.57%	9.50%	7.65%	6.12%	5.15%	4.44%	3.57%
0.5	10.71%	9.14%	7.71%	6.41%	5.25%	4.46%	4.14%	3.41%	2.46%
Panel C: Impact of Net-Worth Provision for American-Style CatEPut									
0.1	100.00%	100.00%	100.00%	100.00%	100.00%	99.95%	96.77%	68.99%	27.30%
0.2	100.00%	100.00%	100.00%	100.00%	99.97%	98.66%	88.52%	47.89%	14.03%
0.3	100.00%	100.00%	100.00%	99.98%	98.55%	89.29%	61.31%	27.95%	8.68%
0.4	100.00%	100.00%	100.00%	99.55%	93.49%	69.27%	32.30%	11.18%	3.19%
0.5	100.00%	100.00%	99.71%	95.96%	76.63%	39.83%	12.69%	3.41%	0.84%

CatEPut ranges from 98,535 to 229,010 (highlighted in red), representing a mere 0.011% to 0.025% of the asset value. However, once the contract is triggered, the capital infusion is also small. This observation highlights the distinctive trading characteristics of contingent capital compared to standard option contracts. Unlike the demand pressure for out-of-the-money options for hedging or speculating purposes (Bollen and Whaley, 2004; Jacobs and Li, 2023), the buyers of contingent capital are primarily focused on the amount of capital injection. This explains why an extremely deep-in-the-money contract is chosen in the RLI paradigm.

Panel B of Table 4 shows that the early exercise premium for the near-the-money CatEPuts ranges from 2.46% to 55.76%. The premium is higher compared to the deep in-the-money case (2.44%–8.98% in Panel B of Table 2) due to the lower price of near-the-money CatEPuts, making them more sensitive to the European-style counterparts. Similarly, Panel C of Table 4 shows consistent findings, with the net-worth provision having a more significant impact on near-the-money contracts than deep-in-the-money ones. For instance, in the RLI example, the early exercise premium falls within the range of 5.59%–11.57% (compared to 4.76%–7.05%), whereas the discount contributed by the provision ranges from 27.95% to 88.52% (compared to 38.12%–92.03% in Panel B of Table 3). The disparity becomes more pronounced for high-leverage and low catastrophe trigger scenarios, for instance, 27.30% compared to

47.53% in the scenario with $L/V_0 = 0.1$ and $\eta = 0.9$. This analysis reveals an unexplored aspect in the literature, as the valuation of near-the-money contracts differs significantly from that of extremely deep-in-the-money contracts, which has not been thoroughly discussed before.

4.3 Bid Prices and Feasible Trading Regions

Here, we first calculate the bid prices for all scenarios discussed in **Section 4.2**. The bid price is defined as the maximum price that decreases neither the firm value nor the equity value. Thus, we analyze the bid price from two perspectives: the firm value maximizers in **Section 4.3.1** and the equity value maximizers in **Section 4.3.2**. To emphasize the bid price analysis from both the firm and equity value angles, this analysis centers on two contract types: (i) European-style `CatEPuts` without the net-worth provision, a fundamental contract widely studied in the literature; and (ii) American-style `CatEPuts` with the net-worth provision, reflecting its application in real-world RLI contracts. We reasonably assume that rational `CatEPut` sellers and buyers avoid detrimental trades. As a result, the deal price must be set lower than the bid price and higher than the ask price to ensure beneficial trades. In **Section 4.3.3**, we consolidate our findings, summarizing the presence or absence of the feasible trading region, represented by the closed interval $[\text{ask price}, \text{bid price}]$, across various contract conditions and provisions.

4.3.1 Perspectives of Firm Value Maximizers

In this analysis, we calculate the bid price from the perspective of a buyer seeking to maximize the firm value. Since the bid price pattern closely resembles the ask price from the previous section, we present the bid-to-ask price ratio in Table 5. A ratio higher than 100% indicates the presence of a feasible trading region, while a ratio lower than 100% indicates its absence. Panel A displays the results for European-style `CatEPuts` without the net-worth provision, and Panel B shows outcomes for American-style `CatEPuts` with the net-worth provision. By examining these ratios, we can analyze the circumstances under which `CatEPut` contracts are more likely to form trading regions.

The bid-to-ask price ratios in Panel A of Table 5 are consistently lower than 100%, whereas those in Panel B are nearly all higher than 100%. These findings emphasize the critical role of contract provisions in forming the trading region. Our untabulated results reveal that even after incorporating the net-worth provision into the European-style contract, the ratios remain below 100%. This finding suggests that the early exercise provision has a dominant impact on the tradability of `CatEPut` contracts.

The particularly low bid-to-ask price ratios for European-style contracts occur when both the leverage ratio and catastrophe trigger level are high. In such cases, the buyer faces a higher risk of insolvency that could invalidate its unexercised `CatEPuts`. Conversely, the early exercise provision present in American-style contracts offers buyers the opportunity to receive a capital infusion upon

Table 5: Bid-to-Ask Price Ratio of `CatEPuts` for Firm Value Maximizers
This table shows the bid-to-ask price ratio under various catastrophe trigger levels (L) and leverage ratios (η), where the buyer seeks to maximize the firm value. Panel A displays the results for European-style contracts without the net-worth provision, and Panel B shows outcomes for American-style contracts with the net-worth provision. The four scenarios that sandwich the RLI case stated in Table 1 are highlighted in red.

L/V_0 \ η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Panel A: European-Style `CatEPut` without Net-Worth Provision									
0.1	99.988%	99.950%	99.798%	99.264%	97.488%	92.874%	81.282%	63.365%	44.743%
0.2	99.982%	99.936%	99.758%	99.145%	97.151%	91.962%	79.513%	59.924%	42.426%
0.3	99.968%	99.899%	99.646%	98.804%	96.169%	89.340%	75.315%	58.405%	42.110%
0.4	99.936%	99.813%	99.386%	98.004%	93.844%	83.552%	69.519%	53.987%	40.190%
0.5	99.861%	99.613%	98.776%	96.129%	88.607%	76.153%	64.567%	49.778%	37.963%
Panel B: American-Style `CatEPut` with Net-Worth Provision									
0.1	100.001%	100.020%	100.111%	100.453%	101.681%	104.956%	107.903%	109.972%	109.251%
0.2	100.002%	100.042%	100.228%	100.939%	103.457%	109.108%	115.722%	120.057%	112.436%
0.3	100.005%	100.094%	100.517%	102.113%	106.399%	114.328%	122.638%	121.342%	110.390%
0.4	100.012%	100.211%	101.162%	104.340%	110.806%	123.267%	131.452%	127.517%	106.046%
0.5	100.028%	100.482%	102.417%	107.191%	116.105%	129.943%	138.512%	131.308%	98.463%

contract trigger, subsequently boosting the firm's value. This enhancement in value increases the buyer's willingness to pay for the `CatEPut`, contributing to higher bid-to-ask price ratios.

An interesting phenomenon occurs when both the leverage ratio and catastrophe trigger level are low: the bid-to-ask price ratios in both panels are quite close to 100%. This suggests that market participants may share common expectations about contract prices. However, the narrowness of this trading region may limit bargaining flexibility, making it challenging to successfully trade the contract under these specific conditions. For the RLI example, the bid prices in Panel B of Table 5 are consistently at least 15% higher than the corresponding ask prices. This notable difference strongly suggests the existence of a trading region, allowing for adequate flexibility in negotiating premiums.

4.3.2 Perspectives of Equity Value Maximizers

Table 6 presents the bid-to-ask price ratio, this time from the viewpoint of a buyer seeking to maximize the equity value. As with Table 5, a ratio above (below) 100% suggests the existence (absence) of a feasible trading region. However, there is a key distinction between these two types of buyers. The capital infusion provided by `CatEPuts` may partially go to debtholders, which can be measured from the firm value perspective but not from the equity value perspective. This nuance may reduce the tradability of `CatEPut` contracts.

The results presented in Table 6 reveal several noteworthy findings regarding the tradability of `CatEPut` contracts from the perspective of buyers seeking to maximize the equity value. First, the bid-to-ask price ratios in both panels are generally lower than those observed in Table 5 (where the focus was on firm value maximizers). This pattern aligns with expectations, as mentioned above, where the capital infusion may partially benefit debtholders but not equity holders. This indicates that `CatEPuts` are more challenging to trade successfully when buyers seek to maximize equity value. Additionally,

Table 6: The Bid-to-Ask Price Ratio of `CatEPuts` for Equity Value Maximizers

This table shows the bid-to-ask price ratio under various catastrophe trigger levels (L) and leverage ratios (η), where the buyer aims to maximize the equity value. Panel A displays the results for European-style contracts without the net-worth provision, and panel B shows outcomes for American-style contracts with the net-worth provision. The four scenarios that sandwich the RLI case stated in Table 1 are highlighted in red.

L/V_0 \ η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Panel A: European-style `CatEPut` without Net-Worth Provision									
0.1	99.982%	99.938%	99.799%	99.384%	98.154%	95.016%	86.559%	68.298%	41.959%
0.2	99.969%	99.896%	99.662%	98.963%	96.909%	91.571%	77.936%	50.014%	24.331%
0.3	99.938%	99.791%	99.320%	97.909%	93.799%	82.986%	59.475%	33.736%	14.799%
0.4	99.868%	99.554%	98.547%	95.528%	86.774%	64.629%	33.799%	14.098%	5.404%
0.5	99.705%	99.002%	96.752%	89.996%	71.041%	38.990%	14.002%	4.495%	1.536%
Panel B: American-style `CatEPut` with Net-Worth Provision									
0.1	99.993%	99.978%	99.933%	99.799%	99.461%	98.532%	96.642%	99.539%	110.864%
0.2	99.987%	99.956%	99.862%	99.584%	98.871%	97.039%	94.776%	100.166%	112.350%
0.3	99.970%	99.900%	99.686%	99.049%	97.467%	95.324%	95.302%	100.599%	113.642%
0.4	99.932%	99.775%	99.293%	97.867%	95.166%	94.173%	96.170%	101.435%	114.432%
0.5	99.844%	99.484%	98.384%	95.827%	93.462%	94.616%	97.275%	102.228%	115.074%

similar to Table 5, the bid and ask prices are closely aligned for scenarios with low leverage ratios, indicating a consensus among market participants.

One of the major findings from Table 6 is its potential to override the conclusions of Table 5. Specifically, the bid-to-ask price ratios for American-style `CatEPuts` with net-worth provisions may become lower than one when buyers seek to maximize equity value instead of firm value. This observation suggests that the buyer's objective function significantly influences the existence of the trading region. These findings support the model setup in this study, the aim of which is to investigate buyers' decisions from different perspectives.

Moreover, the trading region emerges when the leverage ratio is high ($\eta \geq 0.8$) for the American-style contract with net-worth provision. This outcome emphasizes the critical value of considering both the early exercise and net-worth provision in the analysis of tradability. In the case of the RLI example, the trading regions lie within an ambiguous zone between existence and non-existence, with the transaction price falling precisely within the acceptable bargaining range for both buyers and sellers.

4.3.3 Feasible Trading Regions

Table 7 summarizes the existence of feasible trading regions for firm value maximizers (Panel A) and equity value maximizers (Panel B). We consider the presence or absence of net-worth (NW) and/or early exercise (EE) provisions, along with varying leverage ratios (η) and catastrophe trigger levels (L/V_0). The $\cdot$ symbol denotes that the difference between the bid and ask prices is within 1%, signifying a market consensus on the contract price, and the $\surd$ ($\times$) symbol indicates the presence (absence) of the trading region, with $\surd$ ($\times$) denoting the bid price being higher (lower) than the ask price. The number of $\surd$ ($\times$) symbols, up to three, indicates the degree of feasibility of the trading regions.

Table 7: Summary of Feasible Trading Regions

This table summarizes the presence or absence of the feasible trading region under various provisions, leverage ratios (η) in the first row, and catastrophe trigger levels (L/V_0) in the second row. NW and EE represent the net-worth and early exercise provisions, respectively, as illustrated in the first and second columns. Panels A and B show the results for firm value and equity value maximizers, respectively. The $\cdot$ symbol denotes that the difference between the bid and ask prices is within 1%, $\surd$ ($\times$) indicates the bid price exceeding (falling short of) the ask price by 1% to 5%, $\surd\surd$ ($\times\times$) indicates a difference of 5% to 10%, and $\surd\surd\surd$ ($\times\times\times$) denotes a difference of over 10%. The scenarios closely resembling the RLI case in Table 1 are highlighted in red.

Provision		Low leverage ($\eta = 0.2$)			Medium leverage ($\eta = 0.5$)			High leverage ($\eta = 0.8$)		
NW	EE	$L/V_0 = 0.1$	$L/V_0 = 0.2$	$L/V_0 = 0.3$	$L/V_0 = 0.1$	$L/V_0 = 0.2$	$L/V_0 = 0.3$	$L/V_0 = 0.1$	$L/V_0 = 0.2$	$L/V_0 = 0.3$
Panel A: Firm Value Maximizers										
No	No	·	·	·	$\times$	$\times$	$\times$	$\times\times\times$	$\times\times\times$	$\times\times\times$
No	Yes	·	·	·	$\surd$	$\surd$	$\surd\surd$	$\surd\surd\surd$	$\surd\surd\surd$	$\surd\surd\surd$
Yes	No	·	·	·	$\times$	$\times$	$\times$	$\times\times\times$	$\times\times\times$	$\times\times\times$
Yes	Yes	·	·	·	$\surd$	$\surd$	$\surd\surd$	$\surd\surd\surd$	$\surd\surd\surd$	$\surd\surd\surd$
Panel B: Equity Value Maximizers										
No	No	·	·	·	$\times$	$\times$	$\times\times$	$\times\times\times$	$\times\times\times$	$\times\times\times$
No	Yes	·	·	·	·	$\times$	$\times$	$\times\times$	$\times\times\times$	$\times\times\times$
Yes	No	·	·	·	·	·	·	$\surd\surd$	$\surd\surd$	$\surd\surd$
Yes	Yes	·	·	·	·	$\times$	$\times$	·	·	·

Based on the summary results in Table 7, we propose the following findings.[29] First, when the leverage ratio is low (the "Low leverage" column of the table), there is a consensus among market participants regardless of the provision settings. However, insurers usually have high leverage, and the narrowness of this trading region may also limit bargaining flexibility. Second, while a European-style `CatEPut` without the net-worth provision (NW: No, EE: No) is widely studied in the literature, the absence of a feasible trading region persists regardless of changes in leverage ratios, catastrophe trigger levels, or the buyer's perspective. This implies that buyers and sellers cannot find a suitable deal price that simultaneously enhances their benefits, emphasizing the necessity of including provisions in the valuation of this study.

Third, when considering the high leverage ratio (the "High leverage" column of the table) commonly faced by common insurers, significant disagreement emerges among market participants. For firm value maximizers, the bid prices are significantly higher than the ask prices for American-style contracts because the in-time emergency capital infusion increases the firm value. For equity value maximizers, involving net-worth provisions is more crucial to make the contract tradable. In particular, the expenses associated with acquiring these contracts can be minimal due to the presence of low ask prices, as analyzed in Table 3, resulting in a limited impact on shareholders. Furthermore, buyers receiving capital injections under this provision must not have experienced bankruptcy, and the infusion of capital at this juncture is also advantageous for shareholders. In conclusion, our analysis suggests that an American-style CatEPut with a net-worth provision (NW: Yes, EE: Yes) stands as the optimal choice for delivering mutual benefits to both equity and debtholders of insurers, as well as the writers of

[29]These findings remain valid when analyzing the hypothetical near-the-money contracts discussed in **Section 4.2.3**.

CatEPut contracts. This is particularly advantageous for high-leverage buyers ($\eta = 0.8$) with suitable catastrophe trigger levels ($L/V_0 = 0.2$ or 0.3), corroborating the real-world CatEPut transaction carried out by RLI (highlighted in red).

4.4 Changes in Claimholders' Values

In the preceding subsections, we investigated how provisions affect bid and ask prices. Now, we shift our focus to how the purchase of CatEPuts influences the values of claimholders. In **Section 4.4.1**, we equate the Deal to the ask price, the minimum amount the seller demands. If acquiring CatEPuts under this premise boosts the insurer's firm-levered or equity value, it suggests room for potential price negotiation. In **Section 4.4.2**, we position the Deal at the midpoint between the bid and ask prices, reflecting possible negotiation scenarios. For simplicity, we focus on the academically popular yet seldom-seen European-style CatEPuts without the net-worth provision, and the more commonly observed American-style CatEPuts with the net-worth provision, to assess the combined impact of both provisions. We present our results for a representative analysis using the trigger level $L/V_0 = 0.2$ as analogous patterns are observed for $L/V_0 = 0.1$ and $L/V_0 = 0.3$.

4.4.1 Setting the Deal Price to the Ask Price

Table 8 shows the changes in the claimholders' values for the two CatEPut variants: basic CatEPuts without provisions (Panel A) and comprehensive CatEPuts featuring both early exercise and net-worth provisions (Panel B). The first row of the table indicates the leverage ratio of the insurer, and the first row of each panel illustrates the corresponding Deal with respect to different provision and leverage ratio settings. We initially display the alteration in the firm-levered value (ΔV^L) resulting from the CatEPut purchase, followed by a breakdown into changes in equity (ΔE) and debt (ΔD) values. Last, we examine changes in tax benefits (ΔTB), bankruptcy costs (ΔBC), and loss compensation (ΔLC), as discussed in **Section 3.3**.

Table 8 reveals that purchasing basic CatEPuts lacking provisions, as in Panel A, reduces the firm value (e.g., $\Delta V^L < 0$), whereas ΔV^L turns positive for purchasing CatEPuts incorporating both early exercise and net-worth provisions, as in Panel B.[30] This observation aligns with the analyses of feasible trading regions in **Section 4.3.3**, suggesting that American-style CatEPuts with net-worth provisions have the potential to improve the firm values and trading opportunities due to the presence of more favorable trading regions. In contrast, European-style CatEPuts without net-worth provisions do not offer these advantages.

Subsequently, we break down the change in firm-levered value into its constituent equity and debt components. In Panel A of Table 8, it becomes evident that the predominant factor driving

[30] The impacts of the leverage η on the trajectory of ΔV^L are intricate. This complexity arises because high-leverage situations involve lower stock prices, augmented intrinsic value, elevated default risk, and a non-linear relationship with the deal price.

Table 8: Changes in Claimholder Values by Setting `Deal` to Ask Price

This table breaks down the changes in claimholders' values using our revised trade-off theory in Equation (9) for two types of `CatEPuts`: European-style without the net-worth provisions (Panel A) and American-style with the net-worth provisions (Panel B). The leverage ratios are $\eta \in \{0.1, 0.2, \ldots, 0.9\}$ listed in the first row, and the ratio of catastrophe loss trigger levels to the initial firm value is $L/V_0 = 0.2$. The deal price `Deal` is set to the ask price calculated in Section 4.2.

Leverage (η)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Panel A: European-Style `CatEPut` without Net-Worth Provision									
`Deal`	11,466,222	11,634,615	11,802,986	11,971,276	12,139,446	12,307,006	12,472,690	12,213,739	9,347,854
ΔV^L	-2,052	-7,456	-28,573	-102,737	-351,951	-1,045,239	-3,022,977	-6,957,184	-9,539,006
Decomposition:									
ΔE	-3,513	-12,048	-39,789	-123,911	-374,301	-1,032,313	-2,749,331	-5,920,161	-6,083,789
ΔD	1,461	4,592	11,216	21,174	22,350	-12,926	-273,646	-1,037,023	-3,455,217
Revised trade-off theory:									
Δ`TB`	-14	-103	-552	-2,419	-10,356	-34,654	-135,895	-355,466	-557,234
Δ`BC`	1,587	7,204	28,021	100,318	341,594	1,010,585	2,887,082	6,601,718	8,981,772
Δ`LC`	-451	-148	0	0	0	0	0	0	0
Panel B: American-Style `CatEPut` with Net-Worth Provision									
`Deal`	12,356,816	12,525,179	12,693,449	12,861,430	13,026,043	13,090,330	12,287,973	7,828,415	2,604,479
ΔV^L	291	5,192	28,861	120,812	454,365	1,242,511	2,277,838	2,354,750	581,832
Decomposition:									
ΔE	-1,661	-5,532	-17,480	-53,382	-146,321	-384,235	-638,183	12,799	281,283
ΔD	1,952	10,724	46,341	174,194	600,687	1,626,747	2,916,021	2,341,951	300,549
Revised trade-off theory:									
Δ`TB`	33	240	1,276	5,480	22,749	62,941	105,142	52,403	-62,931
Δ`BC`	-528	-5,015	-27,585	-115,333	-431,616	-1,179,571	-2,172,696	-2,302,347	-644,762
Δ`LC`	-270	-63	0	0	0	0	0	0	0

the changes in the firm-levered value is ΔE, which remains consistently negative across all leverage ratios, leading to the overall negative ΔV^L. Moreover, the changes in the debt value ΔD are negative when $\eta \geq 0.6$, further reinforcing the adverse impact on firm value. These findings emphasize the importance of considering early exercise and net-worth provisions for tradability of `CatEPuts` under these circumstances.

In contrast, the American-style `CatEPut` with a net-worth provision yields contrasting outcomes. Panel B of Table 8 reveals that ΔD predominantly influences the change in firm-levered value, leading to a positive impact that strengthens firm value. This finding further explains the presence of the feasible trading region for firm value maximizers, as discussed in **Section 4.3.3**. The change in equity value, ΔE, typically assumes a negative value, except when $\eta \geq 0.8$. These findings align with the infeasible or insignificant trading region observed for equity value maximizers. Although studies such as Lo et al. (2013), suggests that `CatEPut` acquisition can lower the default likelihood for high-risk insurers, our analyses underscore potential challenges in achieving this outcome. Specifically, the purchase of improperly designed `CatEPuts` could adversely affect the firm value or equity value for high-leverage insurers, emphasizing the pivotal role played by contractual provisions.

Following the revised trade-off theory in Equation (9), we delve deeper into the breakdown of changes in firm-levered value, specifically focusing on tax benefits, bankruptcy costs, and loss compensation.[31] Table 8 shows that the most influential factor affecting the impact of `CatEPut` purchase

[31] The impact of changes in loss compensation on firm-levered value is relatively minor; however, it remains essential to

on firm-levered value is bankruptcy costs. For European-style contracts without net-worth provisions, ΔBC exhibits a positive trend that escalates steeply as the leverage ratio increases, exerting a detrimental effect on firm value. Additionally, the change in tax benefit (ΔTB) experiences a decline as the leverage ratio rises. Both of these forces are in accord with the pronounced disagreement in **Section 4.3.3** (represented by the $\times\times\times$ symbol in Table 7). Conversely, ΔTB exhibits positive values, with a sole exception ($\eta = 0.9$) for purchasing American-style CatEPuts featuring a net-worth provision. Furthermore, the reduction in bankruptcy costs contributes positively to ΔV^L. In essence, the purchase of CatEPut under this scenario demonstrates its advantageous nature for both debtholders and equity holders, leading to a mutually beneficial outcome.

4.4.2 Setting the Deal Price to the Mid-Price

An important implication of the revised trade-off theory in Equation (9) is that the deal price Deal may not align with the ask price or the present value of the potential capital injection received by the buyer. To consider potential negotiations, we assume the deal price to be the mid-price, the midpoint between the bid and ask prices. Table 9 presents the changes in claimholders' values using bid prices determined by firm value maximizers. Using bid prices determined by equity value maximizers yields similar results; we ignore the analyses for simplicity.

Under the mid-price assumption, deal prices in Panel A (Panel B) of Table 9 decrease (increase) compared to Table 8. This change occurs because the bid prices calculated in Table 5 are lower (higher) than the ask prices, indicating the absence (presence) of the trading region in these contract conditions. The difference in Deal is more pronounced in high-leverage scenarios due to the disagreement between buyers and sellers identified in our previous findings. Consequently, the negative value of ΔV^L in Panel A of Table 8 is mitigated in Table 9, whereas the positive value of ΔV^L in Panel B of Table 8 is impaired in Table 9.

4.5 Robustness Checks

Here, we provide the robustness checks of our findings to potential concerns. Regarding the early exercise provision, we follow Lo et al. (2013) to assume that the CatEPut buyers exercise their rights immediately when the dual-trigger conditions are met to obtain emergency capital injection. Although this early exercise policy is straightforward and pragmatic, the CatEPut is inherently an American-style option. To account for this, we additionally examine the optimal exercise decision adopted by Wang and Dai (2018) using our TTMJ method. However, our results show that granting insurers additional rights to postpone CatEPut exercises to maximize exercise value does not alter our analyses of provisions and feasible trading regions.

revise the trade-off theory. The 95% confidence intervals for LC before and after the purchase of CatEPut are $(64.5, 121.6)$ and $(40.6, 77.9)$, respectively, with both values being statistically significant and positive.

Table 9: Changes in Claimholder Values by Setting `Deal` to Mid-Price

This table breaks down the changes in claimholders' values using our revised trade-off theory in Equation (9) for two types of `CatEPuts`: European-style without net-worth provisions (Panel A) and American-style with net-worth provisions (Panel B). The leverage ratios are $\eta \in \{0.1, 0.2, \ldots, 0.9\}$ listed in the first row, and the ratio of catastrophe loss trigger levels to the initial firm value is $L/V_0 = 0.2$. The deal price `Deal` is set to the midpoint between ask and bid prices calculated in Sections 4.2 and 4.3.1.

Leverage (η)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Panel A: European-Style `CatEPut` without Net-Worth Provision									
`Deal`	11,465,194	11,630,880	11,788,692	11,920,080	11,966,513	11,812,417	11,195,021	9,766,351	6,656,903
ΔV^L	-1,026	-3,728	-14,286	-51,368	-175,972	-522,551	-1,509,772	-3,471,571	-4,749,473
Decomposition:									
ΔE	-2,487	-8,321	-25,523	-72,829	-201,809	-540,147	-1,472,715	-3,550,960	-3,776,639
ΔD	1,461	4,593	11,237	21,461	25,837	17,596	-37,057	79,389	-972,834
Revised trade-off theory:									
ΔV	1,026	3,727	14,267	51,099	172,608	493,665	1,275,332	2,483,776	2,804,086
Δ`TB`	-14	-103	-551	-2,407	-10,197	-33,165	-120,671	-281,903	-391,498
Δ`BC`	1,587	7,203	28,003	100,060	338,383	983,050	2,664,433	5,673,444	7,162,061
Δ`LC`	-451	-148	0	0	0	0	0	0	0
Panel B: American-Style `CatEPut` with Net-Worth Provision									
`Deal`	12,356,962	12,527,780	12,707,898	12,921,787	13,251,180	13,686,464	13,253,952	8,613,480	2,766,424
ΔV^L	146	2,596	14,430	60,406	227,186	621,352	1,140,204	1,178,219	290,978
Decomposition:									
ΔE	-1,806	-8,128	-31,896	-113,555	-370,324	-975,077	-1,598,745	-761,840	139,650
ΔD	1,952	10,724	46,326	173,961	597,510	1,596,430	2,738,948	1,940,058	151,328
Revised trade-off theory:									
ΔV	-146	-2,596	-14,421	-60,244	-224,749	-596,838	-989,047	-841,442	-169,216
Δ`TB`	33	240	1,275	5,473	22,629	61,587	94,993	26,813	-72,702
Δ`BC`	-528	-5,014	-27,577	-115,178	-429,306	-1,156,603	-2,034,258	-1,992,848	-532,896
Δ`LC`	-270	-63	0	0	0	0	0	0	0

Regarding the catastrophe parameters, we expand our analysis to encompass the sample period from 1950 to 2001, aligning with the maturity date of the RLI's `CatEPut` and alleviating concerns regarding potential shifts in catastrophe frequency. However, the parameter variations are minimal, with the estimated λ changing from 0.61 to 0.58, and the estimated s shifting from 1.09 to 1.08. These subtle parameter adjustments have a negligible impact on `CatEPut` prices, as well as changes in firm-levered or equity values.

Last, we explore the impact of different values for the asset volatility parameter on our analysis. Volatility is a critical factor that significantly influences standard option prices in financial theory. Our initial assumption is $\sigma = 5\%$, whereas other studies consider a range of values: Sundaresan and Wang (2015) and Pennacchi and Tchistyi (2019) use $\sigma = 4\%$, Himmelberg and Tsyplakov (2020) analyze up to $\sigma = 7\%$, Chen et al. (2017) consider $\sigma = 8\%$, and Hilscher and Raviv (2014) extend their examination to volatility levels up to 9%. To ensure the robustness of our findings across varying volatility levels, we depict the relationship between firm value (equity value) and asset volatility within the range $\sigma \in \{0.05, 0.06, \ldots, 0.15\}$, as shown in Panel A (Panel B) of Figure 8. In particular, the solid black curve depicts the values prior to the purchase of `CatEPut`, and the dashed red (dotted blue) curve shows the value after purchasing a `CatEPut` with (without) early-exercise and net-worth provisions. The advantage of incorporating both provisions is obvious regardless of the change in σ. The only distinction is that for equity value maximizers, purchasing `CatEPuts` with the two provisions might yield a slightly positive impact on equity values under the condition of elevated asset volatility. Similar to our previous findings, adopting these two provisions provides the opportunity to enhance the benefits for all participants when trading CatEPuts.

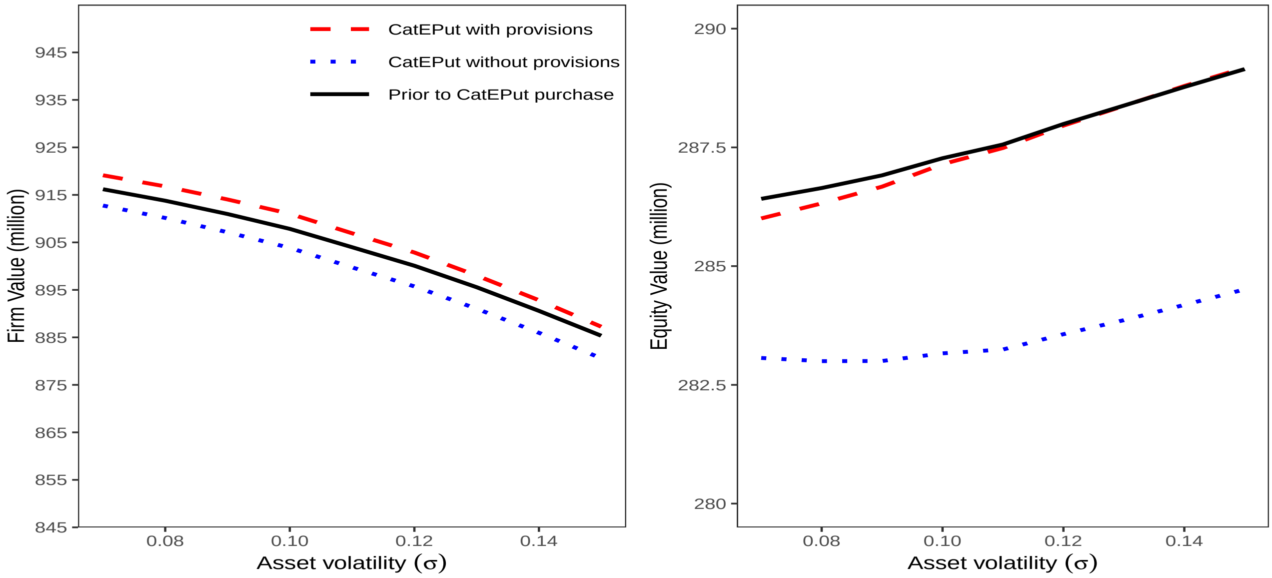

Figure 8: Impact of Asset Volatility on Firm Value and Equity Value. The firm and the equity values before/after purchasing CatEPuts under the asset volatilities listed in the x-axis are illustrated in the left and the right panels, respectively. All other numerical settings follow the descriptions in Table 1.

5 Conclusion

This study examines the impact of contingent capital provisions on acceptable prices for both buyers and sellers. Additionally, it investigates their effect on the values of claimholders to assess the feasibility of early exercise and net-worth provisions. Our model considers catastrophe risk and the buyer's default risk, and addresses an endogenous valuation challenge. Using U.S. catastrophe loss data and a real CatEPut contract acquired by RLI, we quantitatively analyze CatEPut prices from the perspectives of both buyers and sellers. This analysis leads us to investigate feasible trading regions and assess the impact of these contract provisions on firm-levered and equity values.

The main findings are summarized as follows. First, as discussed in Section 4.2, the presence of net-worth provisions significantly lowers the seller's minimum required price for high-leverage firms, and early-exercise provisions increase the seller's required price, especially for low catastrophe trigger levels. Second, as in Section 4.3, we analyze the maximum acceptable price from the perspective of a buyer who maximizes either the firm value or the equity value. We compare the prices acceptable to buyers with those required by sellers, finding that the inclusion of both early exercise and net-worth provisions can create situations that are simultaneously beneficial to the seller, the insurer's equity holders, and the debtholders. Third, we analyze the impact of CatEPut purchase on the change in claimholders' values in Section 4.4. Under different settings of bargaining power among contract parties, we observe that changes in firm-levered values tend to be positive (negative) for `CatEPut` contracts with (without) both provisions, and these shifts are predominantly driven by changes in equity (debt value). Last, based on our revised trade-off theory, we find that the changes in firm-levered value due to the `CatEPut` purchase are predominantly influenced by bankruptcy costs across all contract types. These findings remain robust across various sample periods and levels of asset volatility. Notably, beyond the analyses on `CatEPuts`, the flexibility of our valuation techniques is applicable to assessing the pros and cons of analogous designs in other contingent capital contracts and their pivotal role in catastrophe risk management.

References

Ammar, S. B. (2020). Catastrophe Risk and the Implied Volatility Smile. *Journal of Risk and Insurance 87*(2), 381–405.

Bakshi, G. and D. Madan (2002, March). Average Rate Claims with Emphasis on Catastrophe Loss Options. *Journal of Financial and Quantitative Analysis 37*(1), 93–115.

Black, F. and J. C. Cox (1976). Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *The Journal of Finance 31*(2), 351–367.

Bollen, N. P. B. and R. E. Whaley (2004, April). Does Net Buying Pressure Affect the Shape of Implied Volatility Functions? *The Journal of Finance 59*(2), 711–753.

Braun, A. (2011, November). Pricing Catastrophe Swaps: A Contingent Claims Approach. *Insurance: Mathematics and Economics 49*(3), 520–536.

Brockman, P. and H. J. Turtle (2003, March). A Barrier Option Framework for Corporate Security Valuation. *Journal of Financial Economics 67*(3), 511–529.

Chang, C.-C., S.-K. Lin, and M.-T. Yu (2011, June). Valuation of Catastrophe Equity Puts With Markov-Modulated Poisson Processes. *Journal of Risk and Insurance 78*(2), 447–473.

Chen, N., P. Glasserman, B. Nouri, and M. Pelger (2017, November). Contingent Capital, Tail Risk, and Debt-Induced Collapse. *The Review of Financial Studies 30*(11), 3921–3969.

Collin-Dufresne, P. and R. S. Goldstein (2001). Do Credit Spreads Reflect Stationary Leverage Ratios? *The Journal of Finance 56*(5), 1929–1957.

Cox, J. C., S. A. Ross, and M. Rubinstein (1979, September). Option Pricing: A Simplified Approach. *Journal of Financial Economics 7*(3), 229–263.

Cox, S. H., J. R. Fairchild, and H. W. Pedersen (2004, April). Valuation of Structured risk Management Products. *Insurance: Mathematics and Economics 34*(2), 259–272.

Culp, C. L. (2011, September). *Structured Finance and Insurance: The ART of Managing Capital and Risk*. John Wiley & Sons.

Cummins, J. D. (1988). Risk-Based Premiums for Insurance Guaranty Funds. *The Journal of Finance 43*(4), 823–839.

Cummins, J. D. and P. Trainar (2009). Securitization, Insurance, and Reinsurance. *Journal of Risk and Insurance 76*(3), 463–492.

Dai, T.-S., Y.-D. Lyuu, et al. (2010). The Bino-Trinomial Tree: A Simple Model for Efficient and Accurate Option Pricing. *Journal of Derivatives 17*(4), 7.

Doherty, N. A. (1997). Financial Innovation in the Management of Catastrophe Risk. *Journal of Applied Corporate Finance 10*(3), 84–95.

Duan, J.-C. and M.-T. Yu (2005, October). Fair Insurance Guaranty Premia in the Presence of Risk-Based Capital Regulations, Stochastic Interest Rate and Catastrophe Risk. *Journal of Banking & Finance 29*(10), 2435–2454.

Eom, Y. H., J. Helwege, and J.-Z. Huang (2004, April). Structural Models of Corporate Bond Pricing: An Empirical Analysis. *The Review of Financial Studies 17*(2), 499–544.

Figlewski, S. and B. Gao (1999, September). The Adaptive Mesh Model: A New Approach to Efficient Option Pricing. *Journal of Financial Economics 53*(3), 313–351.

Froot, K. A. (2001, May). The Market for Catastrophe Risk: A Clinical Examination. *Journal of Financial Economics 60*(2–3), 529–571.

Froot, K. A. and P. G. J. O'Connell (2008, January). On The Pricing of Intermediated Risks: Theory and Application to Catastrophe Reinsurance. *Journal of Banking & Finance 32*(1), 69–85.

Gates, B. (2021, February). *How to Avoid a Climate Disaster: The Solutions We Have and the Breakthroughs We Need*. Vintage.

Gatzert, N. and H. Schmeiser (2008, December). Combining Fair Pricing and Capital Requirements for Non-Life Insurance Companies. *Journal of Banking & Finance 32*(12), 2589–2596.

Glasserman, P. and B. Nouri (2016). Market-Triggered Changes in Capital Structure: Equilibrium Price Dynamics. *Econometrica 84*(6), 2113–2153.

Härdle, W. K. and B. L. Cabrera (2010). Calibrating CAT Bonds for Mexican Earthquakes. *Journal of Risk and Insurance 77*(3), 625–650.

Hilscher, J. and A. Raviv (2014, December). Bank Stability and Market Discipline: The Effect of Contingent Capital on Risk Taking and Default Probability. *Journal of Corporate Finance 29*, 542–560.

Himmelberg, C. P. and S. Tsyplakov (2020, October). Optimal Terms of Contingent Capital, Incentive Effects, and Capital Structure Dynamics. *Journal of Corporate Finance 64*, 101635.

Hogg, R. V. and S. A. Klugman (1983, September). On The Estimation of Long Tailed Skewed Distributions with Actuarial Applications. *Journal of Econometrics 23*(1), 91–102.

Jacobs, K. and B. Li (2023, January). Option Returns, Risk Premiums, and Demand Pressure in Energy Markets. *Journal of Banking & Finance 146*, 106687.

Jaimungal, S. and T. Wang (2006, June). Catastrophe Options with Stochastic Interest Rates and Compound Poisson Losses. *Insurance: Mathematics and Economics 38*(3), 469–483.

Koziol, C. and J. Lawrenz (2012, January). Contingent Convertibles. Solving or Seeding the Next Banking Crisis? *Journal of Banking & Finance 36*(1), 90–104.

Kraus, A. and R. H. Litzenberger (1973). A State-Preference Model of Optimal Financial Leverage. *The Journal of Finance 28*(4), 911–922.

Lakdawalla, D. and G. Zanjani (2012). Catastrophe Bonds, Reinsurance, and the Optimal Collateralization of Risk Transfer. *Journal of Risk and Insurance 79*(2), 449–476.

Leland, H. E. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *The Journal of Finance 49*(4), 1213–1252.

Liu, L.-C., T.-S. Dai, H.-H. Chang, and L. Zhou (2022, November). A Novel State-Transition Forest: Pricing Corporate Securities with Intertemporal Exercise Policies and Corresponding Capital Structure Changes. *Quantitative Finance 22*(11), 2021–2045.

Liu, L.-C., T.-S. Dai, and C.-J. Wang (2016, November). Evaluating Corporate Bonds and Analyzing Claim Holders' Decisions with Complex Debt Structure. *Journal of Banking & Finance 72*, 151–174.

Lo, C.-L., C. W. Chang, J.-P. Lee, and M.-T. Yu (2021, September). Pricing Catastrophe Swaps with Default Risk and Stochastic Interest Rates. *Pacific-Basin Finance Journal 68*, 101314.

Lo, C.-L., J.-P. Lee, and M.-T. Yu (2013, December). Valuation of Insurers' Contingent Capital with Counterparty Risk and Price Endogeneity. *Journal of Banking & Finance 37*(12), 5025–5035.

Malamud, B. D., G. Morein, and D. L. Turcotte (1998, September). Forest Fires: An Example of Self-Organized Critical Behavior. *Science 281*(5384), 1840–1842.

Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance 29*(2), 449–470.

Merton, R. C. (1976). Option Pricing When Underlying Stock Returns are Discontinuous. *Journal of Financial Economics 3*(1-2), 125–144.

Pennacchi, G. and A. Tchistyi (2019, June). Contingent Convertibles with Stock Price Triggers: The Case of Perpetuities. *The Review of Financial Studies 32*(6), 2302–2340.

Pennacchi, G., T. Vermaelen, and C. C. P. Wolff (2014, June). Contingent Capital: The Case of COERCs. *Journal of Financial and Quantitative Analysis 49*(3), 541–574.

Pollett, P., H. Zhang, and B. J. Cairns (2007, June). A Note on Extinction Times for the General Birth, Death and Catastrophe Process. *Journal of Applied Probability 44*(2), 566–569.

Sundaresan, S. and Z. Wang (2015). On the Design of Contingent Capital with a Market Trigger. *The Journal of Finance 70*(2), 881–920.

Swiss Re (2018). Sigma 1/2018: Natural Catastrophes and Man-Made Disasters in 2017: A Year of Record-Breaking Losses. Technical report, Swiss Re Group.

Wang, C.-J. and T.-S. Dai (2018, November). An Accurate Lattice Model for Pricing Catastrophe Equity Put Under the Jump-Diffusion Process. *IEEE Computational Intelligence Magazine 13*(4), 35–45.

Zhao, Y. and M.-T. Yu (2020, December). Predicting Catastrophe Risk: Evidence from Catastrophe Bond Markets. *Journal of Banking & Finance 121*, 105982.